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Science is a differential equation
Alan Turing

Glenn F. Webb: A Career in Mathematics

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Let me begin by thanking the editors for compiling this volume honoring the 65 birthday of Professor Glenn F. Webb. It is hard to believe that almost forty years have passed since I first met Glenn at Vanderbilt. It is my distinct good fortune to have had Glenn as teacher, mentor, colleague, collaborator, and friend. In a profession filled with some gargantuan egos, one should always bear in mind the words of St. Augustine: "*It was pride that changed angels into devils; it is humility that makes men as angels*". Glenn has always eschewed the limelight and distained being a center of attention. With Glenn, the message has always been the subject and never Glenn. He always tries to be a partner in the process of learning and discovery. This is readily apparent to those of us who have sat in Glenn's classes, attended his scientific lectures, or collaborated with him. I should say at the onset that this volume has been prepared and this essay written despite the misgivings of Glenn. Were the decision left to Glenn, neither the volume nor the essay would have appeared.

Glenn has authored or co-authored over 140 papers, written one research monograph, and co-edited four volumes. He has given plenary lectures, colloquia, and seminars across the globe, and he serves on the editorial boards of 11 archival journals. He has been the dissertation advisor of sixteen students. The existence of this compiled volume is in itself a testimony to his dedication and pursuit of scientific excellence. As we honor Glenn, we honor what is excellent in our profession.

Glenn was born in Cleveland Ohio, but grew up in Miami, Florida. He attended Georgia Institute of Technology with the original intention of studying chemical engineering. However, he quickly turned to mathematics. He pursued graduate studies at Emory University working under the supervision of John Neuberger. Many of Glenn's graduate courses followed the Socratic methods of the legendary R.L. Moore and H.S. Wall of the University of Texas. Under the Socratic Method, frequently referred to as the Texas Method, students learn fundamental mathematical results through the process of discovery and not from the literature or presentation by the instructor. Although Glenn soon moved beyond the Texas Method, his relentless pursuit of discovery, his independence of thought, and his ability to break complex problems into small manageable parts

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demonstrate his exposure to this methodology during his formative mathematical years. Glenn joined Vanderbilt University in 1968, where he has remained, with the exception of having visiting positions at the University of Kentucky, the University of Rome, Italy, the University of Padua, Italy, Scuola Normale Superiore di Pisa, Italy and Graz University, Austria.

Glenn, along with Michael Crandall, Amnon Pazy, Haim Brezis, Bob Dorroh, Tosio Kato, Jerry Goldstein, and Bob Martin, was one of the pioneers in the area of nonlinear accretive operator theory and nonlinear semigroups and evolution operators, c.f. [1-5, 7-12]. In this setting, nonlinear operators generate nonlinear semigroups and evolution systems. This is not abstraction for the sake of abstraction. The nonlinear semigroup and evolution operators are solution operators of abstract Banach space differential equations, which in turn are function space realizations of nonlinear partial differential equations and other distributed parameter systems. This approach allows one to obtain a well-posedness framework for generalized solutions of these nonlinear systems and to extract qualitative information about these solutions. One particularly important paper of this period of Glenn's career is [5], where he shows that continuous accretive nonlinear perturbations of m-accretive operators produce m-accretive operators. This seemingly abstract result continues to be applied to guarantee the existence of solutions to large classes of nonlinear elliptic, parabolic, and hyperbolic, as well as some higher order partial differential equations. Over the course his career, Glenn has frequently returned to an abstract approach. However, he has always used abstract methods to simplify and illuminate, rather than introducing abstraction for the sake of abstraction.

In 1974 Glenn accepted the prestigious Research Fellowship for Foreign Mathematicians offered by the National Research Council of Italy at the Institute of Mathematics of the University of Rome. As a result of this visit, Glenn not only learned to speak fractured Italian, but more importantly he began an ongoing love affair with Italy and was introduced to the powerful Italian analysis and partial differential equations research community, and had the opportunity to meet a large number of people including Giuseppe da Prato, Ennio de Georgi, Mimmo Iannelli, Vincenzo Capasso, Allessandra Lunardi, Eugenio Sinestrari, and Gabriella di Blasio. Toward the end of his visit he met Rosanna Villella-Bressan, beginning his ongoing collaboration and friendship with her and Janet Dyson [50, 89, 93,103-106,108, 116-117,121-123, 138-140]. About a year before his Italian sojourn Glenn began to consider functional differential equations with discrete or distributed delay. Because the solution operators for functional differential equations of this genre are viewed as mappings of function spaces to function spaces, the mathematical understanding of these operators necessitates an infinite dimensional methodology. Glenn [9] was the first to recognize how nonlinear semigroup theory could be applied to extend the linear theory of functional differential equations developed by Jack Hale and others. A sequence of papers concerning the well-posedness, qualitative properties, and approximation of functional differential equations, Volterra integral equations and integrodifferential equations followed [8-9, 10-20, 22-24]. It was at this time that Glenn began a very successful collaboration with Curtis Travis [13, 15, 19, 27, 26]. Of particular note were fundamental works on semilinear partial differential equations of parabolic type with delays in the nonlinear terms [13], [30]. At the beginning of the second decade of his career Glenn broadened his interest and began to investigate equations with second order time derivatives [25, 27, 30-33, 35-37]. Glenn, working alone and in collaboration with Curtis Travis, soon established himself as leader in the theory of cosine and sine operators , and its application to inhomogeneous and semilinear abstract differential equations and integrodifferential equations with second time derivatives. In many ways this work on inhomogeneous and semilinear hyperbolic and higher order equations paralleled the semigroup approach to equations of parabolic type by Afner Friedman, Amnon Pazy, Jack Hale, Dan Henry and members of the Hale school.

Glenn's work throughout the 1960's and 1970's dealt with generic infinite dimensional systems and not specific equations. With the publication of two papers [38-39] on diffusive epidemic models in 1981 one sees a new emphasis on the consideration of specific applications, in particular mathematical biology to a concentration upon the classical questions of well posedness and the longtime behavior of nonlinear differential equations. These papers on epidemic models formed a basis for Glenn's subsequent interest in the spread of infectious disease [42-43, 80-81, 87, 91-92, 96, 120, 127, 134, 136], some of which was in collaboration with individuals such as Mary Parrott, Bill Fitzgibbon, Jeff Morgan, and Michel Langlais. Early epidemic models frequently used delays to describe periods of disease incubation and latency. A more accurate and natural means of introducing periods of latency or incubation from both the mathematical and modeling points of view is age structure. As a result of what in retrospect proved to be a very important NSF regional conference held at the University of West Virginia, Glenn began to consider the nonlinear age structured population models of Dick McCamy and Mort Gurtin.

Age structured population models became and have remained at the core of Glenn's research effort. I remember a conversation in the early 1980's in which Glenn stated "I am totally fascinated with how things grow". I found this comment rather remarkable at the time but only in retrospect did I fully comprehend how well it would predict the future course of Glenn's research. Population growth became a central theme to Glenn's research, be it populations of cells, viruses, animals, human populations, tumors, infectious agents, genotypes, bacteria, or prions. Because of its centrality to his later work, we provide a short digression on population models.

In the Sanskrit text Bhagavad-Gita of 200 BC, it is written that "Death is sure that which is born as birth is for that which is dead". Population models are predicated on the birth and death processes. The basic linear Malthusian population known to first year calculus students predicts exponential growth or decay of the population $P(t)$ depending upon whether or not the birth rate b exceeds the mortality rate m. Population growth or decay is never purely exponential and dampening effects are incorporated via prescribing logistic regulation, as in the monostable model given by the Verhulst or logistic equation. Bistable dynamics can be described by the Allee effect or other nonlinear controls. More realistic models need to account for the fact both the birth rate and the mortality rate depend upon age as well as elapsed time. This comment leads to inclusion of another independent variable, a, representing age dependent birth and mortality rates given by $b(a)$ and $m(a)$ respectively, and a new dependent variable, $p(a, t)$, representing the time dependent age density of the population. In this scenario, the total population at time t is given by $P(t) = \int_{[0,\infty)} p(a, t)da$. From a practical standpoint one can assume that there exists an $A > 0$ so that $p(a, t) = 0$ for $a > A$. If we make the assumption that individuals leave the population through mortality and that the mortality in the population at time t and age a in a time interval h is proportional to the size of the population and h we obtain,

$$p(a+h,t+h)-p(a,t)=-m(a)p(a,t)h.$$

Taking the limits as $t\to 0$ one obtains the celebrated Sharpe, Lotka, and McKendrick linear age transport equation

$$\partial p/\partial t+\partial p/\partial a=-m(a)p$$

subject to the initial condition

$$p(a,0)=p_0(a),$$

together with an age boundary condition commonly called a birth function of the form

$$B(0,t)=\int_{[0,\infty)} b(a)p(a,t)da.$$

In case the birth and mortality rates depend nonlinearly on time at the total population given by $b(a,t,P(t))$ and $m(a,t,P(t))$ we have a nonlinear age transport equation of the form

$$\partial p/\partial t+\partial p/\partial a=-m(a,,t,P(t))p$$

with the same initial condition and the birth function has a form similar to

$$B(0,t)=\int_{[0,\infty)} b(a,t,P(t))p(a,t)da.$$

If the birth rate and the mortality rate are constant, the linear transport equation and semilinear transport equations may be formally integrated on the age interval $[0,\infty)$ to produce the Malthusian and Verhulst equations respectively. Glenn has frequently modeled epidemics by a system of differential equations with leading equation representing dynamics of the population of susceptible class and the second equation being a linear or nonlinear age transport equation describing the dynamics of the population density, $\theta(a,t)$, of the infective class, where the birth function is calculated using the loss term from the susceptible class created by the infection. Various stages of the infection such as periods of latency, incubation, low infectivity, high infectivity, recovery, etc. are obtained through integration of $\theta(a,t)$ with respect to age over appropriate segments of the age interval. Introducing age structure also allows one to assess the effect of intervention during the course of the infection

Although the basic linear age transport equation is readily solved by elementary characteristics, the nonlinear Gurtin-McCamy equation and it variants are challenging nonlinear hyperbolic equations. An understanding and appreciation of these equations is gained only by through application of sophisticated functional analytic techniques of nonlinear semigroups and evolution operators.

The central theme of Glenn's work during 1980's up through 1990's was nonlinear population models, cf. [42-82, 86]. The majority of this effort was in the area of what one would call classical mathematical biology and it was concerned with epidemic models, cell populations, and tumor growth as well as work concerning the mathematical theory of general population models. Many of the papers during this time frame were singly authored and others were written in collaboration with individuals such as Karl Kunish, Wilhelm Schappacher, Wolfgang Desch, Eugenio Sinestrari, Mats Gyllenberg, and Annette Grabosh. With the exception of the collaboration with Doug Hardin and Peter Takac and a paper on the spread of anthrax and another on the periodicity of cicada emergences, Glenn's attention has been continuous models as opposed to discrete models. One can safely say that with the publication of the seminal monograph, *Theory of Nonlinear Age-Dependent Dynamics* by Marcel Dekker in 1985 Glenn became and for that matter continues to be recognized as one of the experts (if not the leading expert) on the mathematical theory of nonlinear age dependent population dynamics.

Glenn's work with Denise Kirschner on the immune response and HIV [94-95, 97, 99, 102, 119] can be considered pivotal by virtue of not only impact but perhaps more importantly the fact that this signals Glenn's transition to the realm of the mathematics of medicine or biomedical mathematics from more traditional mathematical biology. Traditional mathematical biology involves the development and analysis of general models, the identification of parameters and the classical mathematical analysis of the resulting equations and systems. Biomedical mathematics is concerned with specific as opposed to generic systems or maladies. It is data driven, and it often requires the specification and estimation of parameters, and the validation of the modeling by comparison of simulations with actual medical data . Glenn's work is directed towards specific diseases such as HIV, anthrax, SARS, drug resistant bacteria, and a variety of cancers. It involves a variety of topics including the immune response, agents of infection, treatment strategies, tumor growth, genetic structure, the spread of infectious disease and toxic agents. Not only has he collaborated with applied mathematicians such as Pierre Magal, Marek Kimmel, Laurent Pujo-Menjouet, Mary Ann Horn, Eva Sanchez, Shigui Ruan, Jan Pruss, Hans Engler, Bruce Ayati, Haiping Zhu, Sten Ardal, Jianhong Wu, Pauline van den Driessche, Lin Wang and Ovide Arino, but Glenn has engaged the community of medical scientists and physicians through his collaborations with Marty Blaser, Erika D'Agata, Miles Cloyd, Seema Barjara, Emily Wang, Nicole Bryce, and Carlos Arteaga.

As we look over the path of Glenn's scientific career one observes an evolution of scientific interest beginning with his interest in nonlinear semigroup theory and leading up to his current activity in biomedical mathematics. Aristotle said: We are what we repeatedly do. Excellence, then, is not an act, but a habit. At each stage of his path, we see seminal contributions in the areas of nonlinear semigroups, functional differential equations, infinite dimensional dynamical systems, mathematical population dynamics, mathematical biology and biomedical mathematics. We detect common elements throughout his work:

- Precise identification and description of the problem or model under consideration.
- Thorough referencing of preceding work.
- Clarity and accessibility of exposition.
- Use of elementary methods when possible, coupled with an ability to employ abstract methods when necessitated by the problem.

Glenn's impact extends far beyond the impressive corpus of his work. Glenn, along with a group of young mathematicians that included Curtis Travis, George Reddien, Bob Miura, Charles

Megibben, Matt Gould and Phil Crooke, energized what charitably could be called a regional Mathematics Department at Vanderbilt, and put it on the path to becoming the internationally recognized department that we see today. At Vanderbilt, Glenn immediately revived and energized the analysis program by offering a sequence of popular courses in real variables, functional analysis, and operator theory. He has taught at all levels or the curriculum from elementary calculus courses to advanced graduate seminars. In addition to directing the dissertations of sixteen graduate students, he has served as a mentor to a large number of individuals, including undergraduate students, graduate students, post doctoral fellows, entry level scientists, and mid-career mathematicians. As a result of Glenn's presence a steady stream of visitors has come and continues to come to Vanderbilt. Of particular note is his long standing successful effort to encourage women in the pursuit of mathematical careers. The experience of Professor Emerita Mary Parrott of the University of South Florida is typical. She writes: "With Glenn's encouragement, I applied for and received NSF support to spend the year at Vanderbilt in 1987-88. The time there was very productive and with Glenn's guidance I began the transition to the field of epidemic modeling. This launched a decade of interesting research collaboration in that area."

Glenn is a devoted family man and understandably proud of their success. They share his penchant for intellectual achievement. His wife Aileen, who holds a PhD in organic chemistry and a MD from Vanderbilt, is a practicing ophthalmologist. Their daughter Brantley has recently graduated with honors from Dartmouth University and attends Yale Law School, and their son Travis has begun his undergraduate studies at Brown University. In addition to all this erudite activity, they live a full life with an enthusiasm for athletic activity, the outdoors, and travel.

The academy is a calling as well as a profession. Success is hard won and requires native intelligence hard work, focus, and creativity. Few who pursue this calling will be deemed truly exceptional. Glenn Webb is one of these individuals. I am confident friends and admirers across globe join me in congratulating Glenn on this success and as the Italians say, offer,

"In bocca al lupa",

for his continued success.

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