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Polysystem Modelling of Geographical Processes and Phenomena in Nature and Society

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Abstract. Polysystem methodology elaborated for comprehensive analysis of geographical objects considers them as interrelated systems of different types. Each systematic interpretation of a territorial object is formed as a theory describing this object with a special language used for construction of a certain type of models. This paper proposes new methods to develop geographical models and describes several types of systematic models constructed by these methods.

Key words: polysystem methodology of research and modelling, polysystem, system theories, induction of theories, homology and homotopy of models
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1. Introduction

Geographical science widely uses various models and methods developed in its own framework or coming from other sciences. Since geography is essentially an interdisciplinary science studying natural, economic and social processes and phenomena, there is a vast set of models and they are constantly changing. Geographical models have some specific features which distinguish them from models of other sciences. They should relect the interconnection of different components of the landscape in space and in time. In modern science this is realized by means of geoinformatical modelling. However, even more important is the adaptation of a mathematical model to a specific geographical location, in the other words, identification of the equations of the model. Thus, specific character of modelling in geography depends on its peculiarities and results from its methodology.

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There is a wide spectrum of scientific publications on application of methods of modelling in geography. The outburst of research in this field took place in the 1960-1970s. Many works on this problem were published at that time [1]-[5]. They developed the principles of construction of geographical models which are still of importance and continue to develop on the modern methodical and technological basis. This systematic approach was important for formation of theoretical foundation of geographical science. Development of this scientific trend is summarized in the monograph by A. Trofimov and E. Igonin [6]. It can be considered as a starting point of the development of methods of modelling of geographical systems, as well as a complex of knowledge, which is not yet completely formed.

In the 1970-1980s, laboratories and chair of modelling were created practically in every Russian academic institute and university. However, beginning from the 1990s, they were closed almost everywhere. There are several reasons for this situation.

Firstly, it was impossible to obtain significant scientific results without appropriate professional qualification, without knowing mathematics and without understanding the fundamentals of geographical science. As a result, there prevailed either pure formal approaches or simplest statistic methods of in-situ data analysis.

Secondly, researchers working in geographical sciences were not able to use their knowledge for system analysis and to create mathematical models. These questions require methodological recommendations providing an appropriate mechanism for interpretation of geographical data.

Thirdly, there has been always a deficit of reliable methods to retrieve new knowledge on geographical phenomena from available data. At present time, digital data from space imaging with various resolution partially solves this problem. They allow the development of methods of automated processing of geo-images and creation of maps for special purposes. Remote information gives a possibility to use mathematical models in order to process raster images, to reveal unknown properties and to specify the mathematical models themselves.

Fourthly, it is especially important in modelling in geography to identify models, i.e., to estimate parameters and coefficients of equations taking into account local conditions. Complexity of this task is comparable with complexity of geographical problems themselves. It is even possible to state that identification of models is the main task of geography. It should explain how realization of physical and economic-geographical processes changes from place to place and how these changes affect coefficients of models describing these processes. Geography will achieve its perfection when it will be able to identify situations and to predict their development. In this sense, geography should become a science of high precision.

Finally, in the past, modelling encountered some difficulties in presentation of the results and in creation of the corresponding maps. At present time, due to the introduction of geo-informatical systems (GIS) this problem does not exist any more. Application of geo-informatical technologies allows elaboration of methods of geo-informatical modelling and mapping. GIS-modelling promotes the creation of hybrid systems and of integrated GIS, which bring together databases, systems of data and models. This makes the GIS structure closer to real features of geographical spatially distributed systems. Thus there are currently no are no empirical and technological limitations on the development of methods for modelling geographical systems. At the same time, particular empirical generalization and conceptual ideas become accessible to the scientific com-

munity only after their system organization and presentation in a model understandable to everybody. Computer programmes for automated processing of information are only created after model schematization and algorithmization.

Thus, at present time there are no neither empiric nor technological limitations for the development of methods in modelling of geographical systems. At the same time, particular empiric generalization and conceptual ideas become accessible to the scientific community only after their systematic organization and appropriate presentation. Computer programmes for automated processing are created only after formalization of the model and development of the corresponding algorithm.

Understanding of the role of methodology in geographical modelling is important for formation of geographical science. Development based on this understanding does not limit creation of models in geography but puts a question about geographical meaning of the results. Not only spatial localization or complexity of the objects give geographical importance to the results but also ability of the scientist to make a computational model more concrete, to provide definiteness and engineering accuracy of its realization under specific environmental conditions. Efficiency of this work depends on the quality of initial data, on basic hypothesis, on the methods of modelling and presentation of the results. It makes geographical models as indicator of theoretical and practical importance of geographical studies.

2. Models and methods

Geographical methods are quite powerful and allow the description of various situations. In order to describe them in the form of geographical models, methods of polysystem analysis and synthesis are developed [7]-[9]. Polysystematic character of geographical data turns into well structured theory, which allows the description of geographical objects and of their territorial association. Poly-system project modelling allows the presentation of geographical objects and the explanation of their characteristics using different theoretical languages.

Characteristic feature of geographical studies is their multiple aspects. It is typical for the whole science and particularly for geography and expressed as a pluralism of cognition – philosophical position admitting the existing of numerous equivalent independent and reducible to each other forms of knowledge and methodologies of cognition (epistemological pluralism) or forms of being (ontological pluralism). Usually this feature is associated with the uniqueness of personal experience, subjective, historical and social stipulation of knowledge. However, objective characteristics of the real world are re?ected in the polysystem methodology.

According to this standpoint, geographical phenomena are impossible to “measure” with any single theory, nor is it possible to create a systems theory of geographical content only. To do this requires *reach-through theories* are necessary, each of which would explain in its own system language its monosystem section of objects, sees it from its viewpoint and, on the whole, forms its polysystem image. Every geographical researcher solves problems in his own way, and sometimes it is rather difficult to understand and identify both the method itself and results of its implementation. It is necessary to have a scientific-theoretical system of coordinates making it possible to

arrange any knowledge in the space of all possible theories and project it onto each domain in order to evaluate its novelty, usefulness and development prospects within the framework of each theory. This problem is addressed by polysystem geography.

A polysystem consists of different-quality system layers, nonoverlapping systems; for instance, a polysystem of landscape facies where each facies unit is the manifestation of a definite type (layer) of facies. Similarly, science distinguishes the type of system knowledge corresponding to a definite systems theory. Each theory should be considered as an independent coordinate of the space of knowledge presentation, while a fragment (unit) of knowledge from this theory should be regarded as the value of such a coordinate, or special knowledge about a particular object.. Based on this, a peculiar matrix structure of knowledge organization is generated in the form of "objects $\times$ theories" Cartesian product..

The matrix model of knowledge presentation is a lattice structure in which knowledge about an object is considered (1) within the system of all knowledge about the object (vertically), (2) within the system of all knowledge about the theoretical reach-through layer, the subject of investigation (horizontally). The subject of investigation is the system quality which is considered in the object in terms of a given systems theory, i.e., the laws of formation of systems of a given kind. Objects can overlap, i.e. they can have common components, but they may not overlap if they are sorted into groups (divided). Theoretical sections never overlap, although theories are able to exchange their respective results: knowledge from one reach-through theory can be applied (disseminated) in the space of knowledge of another theory. Knowledge of one reach-through theory cannot be deduced from another. Theoretically, knowledge obeys the laws of a special theory. Object knowledge is very changeable, whereas subject knowledge is stable and is always in a definite place in the scheme of theoretical deduction.

An example of a useful application of knowledge from some theory in another theory is provided by logic without which organization and development of knowledge is impossible. Logic of polysystem analysis is represented by peculiar logic of bundle, vector-combinatory logic, that models the basic principles of Hegelian dialectic triad logic [8]. Logic operates with layers – opposites, i.e. with a set of nonoverlapping systems that generate a polysystem in the form of the coordinate space of knowledge. One layer, the opposite, is the coordinate. Coordinates arise through the use of (1) the operation of negation (generation of a layer of the opposite quality), and (2) the operation of mediation (the birth of a new layer through the synthesis of the opposites). Negation is specified by the (Fig. 1a) $p : A \to B$, where B is a not-A ($\bar{A}$). Mediation arises as the joining from two arrows of different opposites: p_A: $A \to C$, p_B: $B \to C$. Mediation C is A and not-A, i.e. B, simultaneously. The arrows of negation form a communicative diagram (see Fig. 1a), and a mathematical category, the objects of which are the opposites (nonoverlapping systems), whereas morphisms are specified by the operations of negation and diamorphism , i.e. the mirror (isomorphic) mapping of one opposite into the other (identity of the opposites). According to logical diagram, C is negation of B, which means negation of A. Therefore, C is a negation of A. Such logic is intrinsically contradictory, but it is natural and convenient because it operates with only one logic invariant, the truth, which is preserved in the case of diamorphic mapping, i.e. at the transition from the action space of one opposite to the space of the other opposite (from layer to layer).

In vector-combinatorial logic, negations are differently directional, like the coordinate of space (Fig. 1b), and in the case of negation the truth of the statement . This is one of the rules of conclusion. The second inference rule is concerned with the operation of mediation: if two initial opposites are true simultaneously, then their mediation is also true. Simply stated, if one of the objects of the communicative diagram is true, then all the other objects are also true.

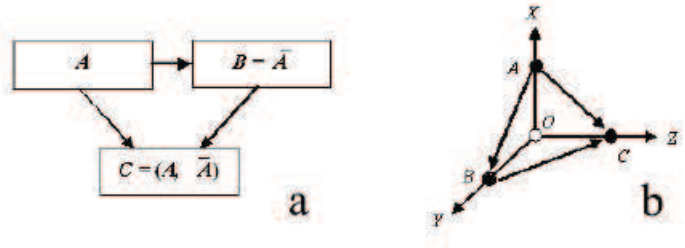

Figure 1: Communicative scheme (a) of relationship of thesis (A), antithesis (B) and synthesis (C) and logic space (b) forming their relations in the orthogonal system

This logic is more appropriate to the style of substantive scientific-geographical thinking in the multidimensional space of diverse opposites. One can do with Aristotelian formal logic in the one-coordinate space. Vector-combinatory logic is used not only in philosophy (quantity *A* – quality *B* – measure *C*) but also in any other science, such as geography (nature *A* – population *B* – economy *C*). Logic answers the main questions: Where do the opposites come from? How are the bases for reality bundle formed? How is the analysis space formed? Etcetera.

Theories in their new interpretation are logically contradictory; therefore, they generate inside themselves a multidimensional space of presentation of their own knowledge, within the framework of which the theoretical process of ordering and derivation of new knowledge is realized. Each theory is systematic and a reach-through one, i.e. it explains in a definite systemic language (interpretations) the processes occurring in each object of the reality, whether they are living or nonliving, natural or social structures at different levels of their hierarch-cal organization. The internal multidimensionality of th space of a theory is largely associated with the existence of different-quality objects and the related forms of motion.

Each theory has three axioms: two axioms of existence of a universal system S of a given kind and its universal change ΔS , and one axiom that holds true both for the universum and any other system of a given kind and relates any change ΔS_i to the action D_i that causes such a change.. Each theory expressed its knowledge in appropriate mathematical terms, and there is no such theory which would use a similar mathematical apparatus.

Not any presence of the object but some invariant quality of the system that remains during mappings and transformations, such as the truth or the speed of light that are constant in any logical and physical frames of reference, can be considered as existence. With such a formulation, to exist means to be identical to this invariant C and possess in full measure this invariant quality.

In accordance with these statements, three axioms hold true for any system of this type, its changes and actions:

$$1)S \equiv C, 2)\Delta S \equiv C, 3)\Delta S_i \equiv D_i, \tag{2.1}$$

These are the axioms from the general systems theory which is considered as a mathematical model of philosophical ontology (dialectics) in its materialist treatment. In the substantive sense, they signify: the World exists $S \equiv C$, the World is changing $\Delta S \equiv C$, and a change of any system is engendered by action, i.e. by the struggle of opposites.

The notions and axioms of any other theory constitute interpretation (in the language of this theory) of the basic notions and axioms of the general systems theory in the form (2.1). Several tens of axiomatic systems theories have been created and are being developed further in applications to geographical research to date [8, 9], and for each system its own ontological system of mapping and research [9-11] has been created, with its the core represented by a system of basic notions and axiom of (2.1).

Let us consider an example of several theories which engender types of models of geographical content [8, 9] and are discussed in the other papers of this issue.

2.1. Theory of integrated science

The theory of unified science systematizes the aforementioned statements concerning the relationship of knowledge and theories. Unified science E combines all possible knowledge $e_j \subset E$ which stratifies into various system theories T_i , and these are equivalent through interpretation of notions and logical inference (the identity $\equiv$). Each theory T_i includes knowledge of only a definite type $e_{ij} \subset T_i$. Scientific action corresponds to experiment D_j in the broader sense, including observations, field and model experiments, computational experiment as well as reasoning with the resulting emergence of new knowledge Δe_j . In science, invariant existence is treated as the property of completeness P of a theory, i.e. the invariant quality of such a theory, in terms of which any knowledge can be derived from its axioms by means of logic, and such knowledge can be formulated in the language of a given theory.

Integrated science combines knowledge, theories and methods of theoretical and empirical cognition and it is by itself a theory with the following system of axioms which hold true for any knowledge:

$$1)E \equiv P, 2)\Delta E \equiv P, 3)\Delta e_j \equiv D_j, \tag{2.2}$$

These axioms state that unified science is complete, its change, i.e. any new theory, is complete and any new knowledge is the result of experiment interpretation.

Geography in its modern interpretation supplements unified science in the sense that it uses its theoretical and methodological results to explain the manifestation patterns of territorial objects. Unified science in the "objects $\times$ theories" matrix scheme synthesizes knowledge according to subjects (theories), whereas geography does it according to objects. On the other hand, knowledge from geography forms part of knowledge of unified science and is concentrated in it in the area of describing planetary laws of natural, economic and social phenomena.

2.2. Theory of control mechanism systems

This theory explains mechanical processes ranging from elementary particles to the Universe, but it is also used in modelling different kinds of control mechanisms *in situ* and *in vitro*, and in the economic and social spheres. Thus its "reach-through" character reveals itself. The theory studies objects in terms of the deviation of values of their characteristics from equilibrium ones. Such an approach is convenient because all measurements are made in relative, rather than absolute, values of $\boldsymbol{R} = \{R_i\}$. The universum of this theory is the Universe as a whole with a characteristic radius R, and with the rate of change of this radius dR/dt. The invariant of existence is connected with the speed of light c , and identity is interpreted as an equality within the accuracy of the coefficients of proportionality (k, H). Eventually the action is regarded as the effects of some relative values on the others.

Let the interpretation of the axioms (2.1) in these terms, first for the Universe as a whole, be written as:

$$1)\boldsymbol{R}\text{=}kc,\ 2)d\boldsymbol{R}/dt\text{=}c,3)dR_i/dt\text{=}HR_i, \tag{2.3}$$

Axiom 1 defines the ultimate distance between points in the Universe, and axiom 2 defines the ultimate rate of change of this distance. Axiom 3 is known as the Hubble law for the expanding Universe.

Axioms 1 and 2 contradict each other, because axiom 1 states a constancy of the distance R, i.e. its change is zero (no change), and axiom 2 states a the constancy of the rate of change of the value of R. This is in agreement with the principles of logic described above: each of the axioms specifies its independent and true vector of problem solving. Prior to A.A. Fridman's cosmological publications, axiom 1 was thought to be true, whereas modern astrophysics adheres to model 2.

In addition to the variants (1 and 2) of existence of the Universe, several other variants are possible, which follow from the third axiom for $R : dR/dt = HR$. Let us differentiate both sides of this equation with respect to time $d^2R/dt^2 = HdR/dt$ and substitute its expression $dR/dt = HR$: $d^2R/dt^2 = HR$ for the first derivative. We obtain the statement that contradicts axioms 1 and 2, i.e. a new axiom for the formulation of yet another independent vector of the analysis. The possibility for such transformations is also realized for any relative value, and for a set of relative values of $\boldsymbol{R} = \{R_i\}$ for which axioms in the general form hold:

$$1)\ \boldsymbol{R} = k\boldsymbol{k}_0R, \quad 2)\ d\boldsymbol{R}/dt = \boldsymbol{k}_0c, \quad 3)\ d^r\boldsymbol{R}/dt^r = \boldsymbol{H}\boldsymbol{R}, \tag{2.4}$$

where $\boldsymbol{k}_0$ is a vector of the dimension and scale factors, $\boldsymbol{H}$ is a matrix interaction factors of values; $r(i)$ is the order of derivative, of its own for each value. The order r assumes only four values $(r = 0, 1, 2, 3)$ which is responsible for the existence of four kinds of physical interactions, types of neural activity in behavioural psychology, physical, ecological, economic and social components in geography systems etc. Different-order variables can be involved in the interaction. According to axiom 3 of (2.4), this interaction is described by the system of differential equations:

$$\frac{d^iR_i}{dt^i} = \sum_{j=1}^{n} a_{ji}R_j, \tag{2.5}$$

where $H = \|a_{ji}\|$ is a matrix of interaction factors. The system of equations for interaction between single-order variables is

$$\begin{array}{l} \frac{dR_1}{dt} = a_{11}R_1 + a_{21}R_2 + a_{31}R_3, \\ \frac{dR_2}{dt} = a_{12}R_1 + a_{22}R_2 + a_{32}R_3, \\ \frac{dR_3}{dt} = a_{13}R_1 + a_{23}R_2 + a_{33}R_3. \end{array} \tag{2.6}$$

In the general case, the interaction factors $a_j i$ are variables. Therefore, the right-hand side of equations (2.5) and (2.6) involves bilinear forms of presentation of the action, which lends flexibility ad universality to the models.

The factors a_{ji} can remain in some reference frame tied to with the domain of existence of the model object whose size is specified by axiom 1. Beyond this domain the object changes into a new state to become a new object.

Let us consider, as a particular case, the model of G. Khilmi [12] for the increase in stock volume (m^3 / he $\sim$ m) of normal stands of different site classes:

$$\frac{dR_i}{dt} = \beta(R_{0i} - R_i), \tag{2.7}$$

where R_{0i} is a maximum stock volume (constant), and β is the species coefficient. The difference $\bar{R}_i = (R_{0\text{i}} - R_i)$can may be regarded as a relative stock volume, which brings equation (2.7) to $\frac{d\bar{R}_i}{dt} = -\beta\bar{R}_i$ corresponding to axiom 3 from (2.3) at $H = -\beta$. The equation presents the data on growth trend in yield tables, the coefficient b does not depend on site index but differs in species. R_{0i} increases linearly with increasing site index (from V to the best I).

The value of the maximum stock volume in this case, $R_0 i$, can be regarded as a limiting size; then the limiting rate of change will be $V_{0i} = \beta R_{0i}$. The dimension coefficient $k_{0i} = R_{0i}/R$ from axiom 1 in (2.4) depends linearly on site class (environmental quality). It is evident from this example that from the formal viewpoint the process of expansion of the Universe and the changes in the stock volume follow the same laws whose parameters vary with the scale of the phenomenon and with environmental conditions.

The theory of regulation mechanisms engenders a special class of models which can be widely used in physical as well as economic geography. Self-regulation of geosystems is expressed as a stabilizing dynamics corresponding to present concept of homeostasis [13]. Stabilizing dynamics promotes support of species and generic attributes of facies and geoms, in spite of the external effects. Homeostasis is a crucial condition for the recovery of natural resources and of the properties of environment, e.g. purification of water and air masses, and reproduction of biomass of vegetation cover. A similar approach is used in ecologo-economic modelling [14], and in the games theory. Models of from the theory of mechanisms are convenient, as they can be used to relatively easily pose and solve problems of assessing stability of systems because with their help one can easily set and solve tasks on estimation of system sustainability, optimize management, etc.

2.3. Theory of dynamic systems

This theory deals with the study and modelling of processes of transition of geosystem elements from state to state. It investigates fluxes of matter and energy, recovery/age processes in vegetation cover, the evolution of landscapes, production technology, changes of social indices of the status of society, etc. It is an extensive sphere represented in geography by the theory of geosystems: dynamic component systems with ongoing processes of matter and energy metabolism and long-term evolutionary changes [13]. The input-output balance in value and material terms for economic districts and inter-district economic interactions is also studied in this aspect.

The main concept of this theory is "state" which exists in two aspects: basic and current. The basic characteristic of state Q_i involves the indices of the numerical axis in ordination space where each object is parametrized by a great number of its characteristics Q_i. The current characteristic of the state of a particular object $q_i(t)$ is a variable changing over time and space under the influence of various factors. The location of the object in the basic ordination space of states is identified from the current characteristic of state is sued to characterize . The difference here is the same as between time and age, the altitude of a particular location above the sea level and the height in general, as well as between the value of the particular goods and the value in general, in monetary terms. In this case, time is a current (quantitative) characteristic and age is a basic (qualitative) characteristic. The qualitative characteristic can be measured using an appropriate measure.

Special axioms of this theory are:

$$1)\ Q = kc, \quad 2)\ \frac{dQ}{dt} = c, \quad 3)\ \frac{dQ_i}{dt} = \frac{dq_i}{dt}. \tag{2.8}$$

Here, the universal characteristics of the state of the Earth's natural resources (Q) are constant, and the tendency for a change of the global economy (dQ/dt) is constant; hence the exponential population increase and improvement of the quality of life ($dQ/dt = kQ$), especially s regards its information saturation. Axiom 3 postulates the well-known philosophical principle of transition from quantitative changes $dqi/dt = V_i N_i$ of system elements $N_i(t)$ in state i to qualitative changes $dQ_i/dt = \Delta Q_i I_i$. Here $V_i(t)$ stands for the mean quantitative temporal changes of system elements $N_i(t)$; ΔQ_i is a measure of the necessary changes for transition from the *i*-th state to a next state; and $I_i(t)$ is the flow of elements from the *i*-th state to a next state.

The basic equations from the dynamic systems theory that follow from these statements characterize the balance of the flows of elements from state to state:

$$\frac{dN_i}{dt} = \sum_{j=1}^{n} \alpha_{ji} N_j - N_i \sum_{j=1}^{n} \alpha_{ij}, \tag{2.9}$$

where $\alpha_{ji}(t) = p_{ji}(t)V_i(t)/\Delta Q_i$ is the transition intensity of elements from the *j*-th state to the *i*-th state, and $p_{ji}(t)$ is the fraction of elements issuing from i along the *j*-th direction. The right hand side of equation (2.9) involves the bilinear form of action consisting of a set of the number of elements $N = N_i$ in different states and of a set of variable values of the coefficients $a_i = a_{ji}$, the values of which depend on the properties of elements and impact of environment $V_i(t)$, the structure of the process $p_{ji}(t)$, and on the characteristic of state ΔQ_i.

The structure of the process can be unidirectional and multidirectional (fluctuational) [15], with an increase, acquisition and loss of elements. When the value of the measure is small, i.e. when the state is changing continuously, equation (2.9) is represented as a partial differential equation. For example, the age dynamics of forests dominated by the *i*-th species of age τ occupying at time *t* an area $S_i(t,\tau)$ within the forest territory is described by the equation representing unidirectional age changes in the forest area with succession of species [8]:

$$\frac{\partial S_i\left(t,\tau\right)}{\partial t}+\frac{\partial\left[p_i\left(t,\tau\right)S_i\left(t,\tau\right)\right]}{\partial\tau}=-\gamma_i\left(t,\tau\right)S_i\left(t,\tau\right)+I_i(t,\tau) \tag{2.10}$$

where $\gamma_i(t,\tau)$ is the transition intensity from forests dominated by the of *i*-th species of age τ to forests of other species at time *t*; $p_i(t,\tau)$ is the probability that there will be no successions of species in forests dominated by the of *i*-th species of age τ at time*t*; $I_i(t,\tau)$ corresponds to the total rates of transition from forests of different species composition and uneven ages to forests of the *i*-th species of age τ at time t.

2.4. Theory of systems of potentials

This theory is concerned with potential characteristics of systems, the system of characteristics-potentials$x_i=\{x_{ij}\}$, *i*=0,1,2,...*n*, among which the potential, referred to as the entropy$x_{0i}=S$,the analogue of time, is distinguished. A change in the entropy is just the reverse of a change of the amount of information *H* in the system:$dS=-dH$. Entropy is the measure of disorder, chaos. Information, on the contrary, characterizes order and informativeness. The potential characterizing the system of potentials as a whole is called organization of the system, $W_i(x_i)$. In physics, the internal energy of the system corresponds to the quantity $W_i(x_i)$, and, in a broader sense, the organization $W_i(x_i)$ is treated as the ability of the system to perform any work, i.e. its utility. In such a treatment, the theory of potentials embraces in a "reach-through" manner the phenomena ranging from physical thermodynamics to the description of biological, economic and social organizations. Changes of the organization occur in the space of entropy (information): $T_i=\frac{\partial W_i}{\partial S_i}$. This quantity is called temperature in thermodynamics and investments (changes of the organization as information becomes available) in economics.

There exists some closed system of potentials *W*(*x*) combining all organizations and potentials and its changes *T*. A system of axioms is developed for *W*(*x*), its changes, and for changes of any potential

$$1)\ W=k_0c,\quad 2)\ T=c,\quad 3)\ -kT_i=D_i, \tag{2.11}$$

where *k*, k_0 are constants, and the action is specified by a bilinear function of extensive x_{ij} and intensive $a_{ij}=\frac{\partial W_i}{\partial x_{ij}}$ potentials:

$$D_i=\sum_{i=1}^{n}a_{ij}x_{ij}. \tag{2.12}$$

Entropy and information are absent in the action. The last axiom yields computational equations:

$$-kT_i = \sum_{i=1}^{n} a_{ij}x_{ij}. \tag{2.13}$$

In self-developing organizations$-kT_i = W_i$, and the action coincides with the organization itself, consequently, the dependence $W_i\ (x_i)$ is described by the Euler equation:

$$W_i(x_i) = \sum_{i=1}^{n} \frac{\partial W}{\partial x_{ij}} x_{ij}. \tag{2.14}$$

The solution to the equations developed above is provided by numerous functions: generalized equations of allometry in geography and biology, and production functions in economics help solve the equations enumerated.

2.5. General theory of complex systems – complexes

This theory is formed in mathematical terms of theory of sets, categories, functors and toposes. It reflects the functional connections of components of geographical systems, the connections of geosystems themselves in landscape, the connections of landscapes with one another, etc. Combinations of different elements, components, systems, complexes and systems of complexes result in different compositions and systems of compositions (configurations)X_i. The universal structure X contains all X_i and is, in this sense, similar to the concept of the set of all subsets or categories of all subcategories.

The connections of the compositions are specified by a mapping (morphism) $F_{ij} : X_i \to X_j$. Compositions, connected by morphisms, i.e. representing in the mathematical sense a category, form a complex. A comparison of two compositions X_i, X_j, namely, $\Delta X_{ij} = X_j/X_i$, shows how the *j*-th composition differs from the *i*-th composition. For example, X_i, X_j are different seasonal development stages of landscape; hence ΔX_{ij} a complex of all the current changes (process). If X_i, X_j are two neighbouring landscapes, ΔX_{ij}fixes their territorial differences, and the function F_{ij} expresses the ralation of comparison. Structure ΔX_{ij} is also a composition and belongs to the set of all comparisons D*X*. The set of all possible mappings of comparison F_{ij} is combined in F.

The standard of comparison *I*is formed – a unit segment [0, 1] with the metrized, linearly ordered, continuous (uninterrupted) set of points bounded above and below. One-to-one correspondence of compositions and standards is designated by the relation ($\leftrightarrow$). For example, $I \leftrightarrow I$ means that each point *I* turns into itself, $X \leftrightarrow I$ is any composition from set *X* corresponding unambiguously to a certain point from the unit segment [0, 1] and only to it. In the last case, it turns out that *I* is a bundle base *X*, i.e., differentiation *X* on the composition is carried out by comparing them with points (numbers) out of the unit segment [0, 1].

Axioms from the theory of complex systems would compare compositions and functions of their connection with the standard of the order and with each other:

$$1)\, X \leftrightarrow I;\ 2)\, F \leftrightarrow I;\ 3)\, \Delta X_{ij} \leftrightarrow F_{ij}. \tag{2.15}$$

Axioms 1 and 2 transfer all their properties of the order of the set I to the set of all compositions $X_i \subset X$, their comparisons $\Delta X_i \subset X$ and mappings $F_{ij} \subset F$. Therefore, all these combinations are functors, i.e., structures with an individual measure from I which are linearly ordered with respect to each other. Their comparisons and mappings are unambiguously connected with each other through structures I:

$$X \leftrightarrow F \leftrightarrow \Delta X \leftrightarrow I. \tag{2.16}$$

This emphasizes one important point of interconnecting: complexes, their comparisons and mappings are equivalent and have one identification index from I. Complexes are systems of self-development in which any change is based on their own structure: $\Delta X \leftrightarrow X$. Transitional states of systems with unformed structure of connections cannot be referred to complexes.

Complexes are linear sequences of morphisms (categories)

$$\cdots \rightarrow X_i \rightarrow X_j \rightarrow X_k \rightarrow \cdots, \tag{2.17}$$

therefore, the compositions involved in it form a homological series of comparison, but since each composition has a measure from I, this series is also a homotopic one.

Any fragment of the homological series $X_i \rightarrow X_j$ is a complex to which a comparison complex of the differences $\Delta X_i \rightarrow \Delta X_j$ corresponds. Consequently, a similarity in the structure indicates a similarity of the changes, and vice versa. Thus the similarity observed in the structure must be caused by the similarity of processes and their models. The main subject of the theory of complexes involves searching for structural and dynamic similarity. Of importance for modelling is its main result – all models of well-established geographical complexes are homologically and homotopically similar, i.e., they are able to turn into each other under a changes of identification index – the image of the model or a point on the unit segment I. This conclusion plays an important role in model calibration, and in adjustment of the model to the particular spatial situation.

3. Results

Dendroclimatic data used for indication of different processes have long become the main source of information on geographical phenomena covering a long period of time [16]. Determination of the relationship between physical-geographical characteristics, and climatic characteristics in particular, and dendrochronological data is complicated by the integral manifestation in the ecosystem of: 1) multilevel factors of ambient environment; 2) different-quality laws, and 3) the individual properties of objects-indicators. To obtain an accurate idea of regional climate changes, it is necessary to take into account all these effects and abstract from them using algorithms based on sophisticated mathematical models (polymodels) of bundle of diverse conditions. As a result, such work must involve comparing the data series for differently located objects in order to obtain dendroclimatic indices in a refined form. Are these differences serious enough to distort the climatic signal? There arises a problem of checking similarity of dendroclimatic series of individual objects by the criterion of their homology and homotopy.

The objects of dendrochronological investigation were represented by time series of annual diameter increments of larch trunks for northern areas of Krasnoyarsk Territory (data from the V.V. Sukachev Institute of Forest SB RAS).

Data processing uses, as the base model, the dynamic model relating the current tree diameter increment *D'(t)* (growth rate) to the value of the diameter *D(t)* and the effect of different (*i*) environmental factors $x_i\ (t)$ and with ecological characteristics x_{0i} of trees of a given species [9]:

$$\frac{d\ln(D/D_m)}{d\ln t} = A\ln\frac{D}{D_m} + \sum_i \alpha_i \ln\frac{x_i(t)}{x_{0i}} + \ln B_0, \tag{3.1}$$

where D_m is a limit tree diameter, α_i are weight coefficients, and *A*, B_0 are constants.

The differential equation (3.1) is a polymodel which combines the formal description of tree diameter increment and the functional description of the influence of a set of factors on this process. To separate out the climatic signal proper, the biological and climatic components of the increment *V* are singled out:

$$V(t) = \frac{d\ln(D/D_m)}{d\ln t} - A\ln\frac{D}{D_m} = \sum_i \alpha_i \ln\frac{x_i(t)}{x_{0i}} + \ln B_0 \tag{3.2}$$

With this end in view, a regression analysis is carried out throughout the age interval the relation

$$\frac{d\ln(D/D_m)}{d\ln t} = A\ln D - B,\ B = A\ln D_m \tag{3.3}$$

and the *A, B* and $\ln D_m = B/A$ are determined. With a knowledge of these values, the value of $V(t) = \sum_i \alpha_i \ln x_i(t)/x_{0i} + \ln B_0$ is determined for each time series using the right-hand side of equation (3.2) thereby singling out the climatic signal varying over time (Fig. 2). It depends on the type of local conditions (α_i), the effect of various environmental factors (x_i), the threshold characteristics of tree species (x_{0i}), and on the neglected background effects (B_0).

The moving average method was used to smooth the high-frequency oscillations within an 11-year interval. The tendencies of environmental changes, shown by different indicators, are in agreement, which makes it possible to single out one (the most representative) of the series $V_{c0}\ (t)$ as a standard and to compare the other $V_{cj}\ (t)$ using the formula

$$V_{cj} = a_j V_{c0} + b_j, \tag{3.4}$$

where a_j, b_j are individual coefficients which are statistically reliable at the level not lower than 0.95. These coefficients are correlated: $b_j = \alpha a_j + \beta,\ \alpha = -0.20,\ \beta = 0.115,$ *R*=-0.97. As a result, equation (3.4) represents a bundle of lines

$$V_{cj} - 0.115 = a_j(V_{c0} - 0.20) \tag{3.5}$$

centered on $V_{cj} = 0.115, V_{cj} = 0.20$.

Thus the increment of different trees responds in a correlated fashion to the influence of the ambient environment. This makes it possible to standardize, using formula (3.5) and the identification parameter a_j for each case, the initial series, i.e., to bring them to a standard:

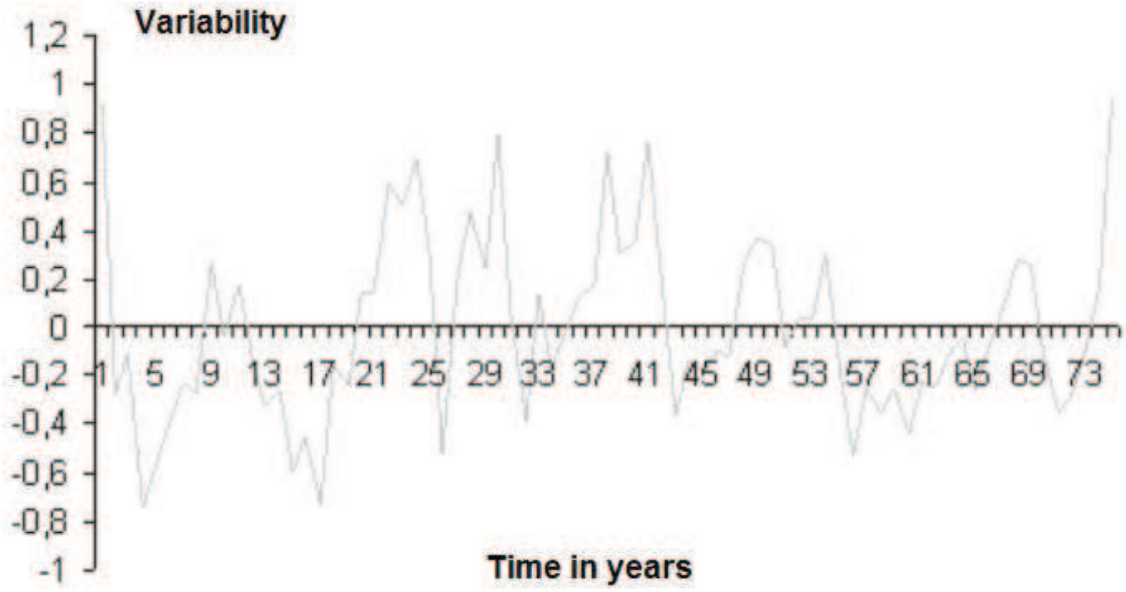

Figure 2: Index fluctuations of environmental changes $V(t)$ within the time

$V_{0j}(t) = (V_j(t) - 0.115)/a_j + 0.20$. Upon averaging the calculated values for different objects over the years, we obtain a dendroclimatic index reflecting the fluctuations in environmental conditions. The similarity of the corrected values with this index is at a rather high level $R = 0.90$. based on calibration done in this manner, the different time series are brought to a single standard series. Each particular series is its variation according to the identifiera_j. It is associated with the fluctuation of local factors or with an individual cyclicity of ecosystem activity.

A constancy of the position of bundle centre (3.5) points to an important fact, namely, stability for the period of observation of global environmental conditions, at the background of which regional factors are at work.

There exist four levels determining the type of dendroclimatic series: general global constant climatic background, general regional variable climatic impact, local factors of microclimate variability and the peculiarities of the response of the particular organisms to the influence of the factors. In these multilevel conditions, a homology of the time series is ensured, which is governed by a similar response of organisms to impacts.

This model can now be used to study the mechanism that is responsible for the homology of series. It is established that there is a linear relationship between the values values $V_j = a_j V_0 + b_j$, where

$$V_j(t) = \sum_i \alpha_{ij} \ln \frac{x_i(t)}{x_{0ij}} + \ln B_0, V_0(t) = \sum_i \alpha_i \ln \frac{x_i(t)}{x_{0i}} + \ln B_0,$$

whence

$$\sum_i \alpha_{ij} \ln \frac{x_i(t)}{x_{0ij}} + \ln B_0 = a_j \sum_i \alpha_i \ln \frac{x_i(t)}{x_{0i}} + a_j \ln B_0 + b_j. \tag{3.6}$$

This relation is of fundamental character if it does not depend on time and on time-dependent factors $x_i(t)$. This is possible if the weight coefficients of different series satisfy the equality $\alpha_{ij} = a_j \alpha_i$, i.e., they vary with a change of the identification index, which means a dependence of the weight of each factor on local conditions a_j, which is natural. Then (3.6) becomes

$$-a_j \left(\sum_i \alpha_i \ln \frac{x_{0ij}}{x_{0i}} + \ln B_0 \right) + \ln B_0 = b_j, \tag{3.7}$$

which determines the observed linear dependence of the coefficients a_j, b_j provided that

$$\sum_i \alpha_i \ln \frac{x_{0ij}}{x_{0i}} + \ln B_0 = -\alpha = const. \tag{3.8}$$

This is achieved when the threshold values for the objects under different local conditions $x_{0ij}/x_{0\mathrm{i}} = c$ are constant. Then

$$\sum_i \alpha_i \ln \frac{x_{0ij}}{x_{0i}} + \ln B_0 = \ln c \sum_i \alpha_i + \ln B_0 = c_0 \ln c + \ln B_0 = -\alpha, \sum_i \alpha_i = c_0,$$

$$b_j = -\alpha a_j + \beta, \quad -\alpha = c_0 \ln c + \ln B_0, \ \beta = \ln B_0. \tag{3.9}$$

From this it is clearly seen that the coefficients $-\alpha$ and β differ by $c_0 \ln c$. If the ratio of the threshold values $x_{0ij}/x_{0\mathrm{i}} = c = 1$, there is an agreement between -a and b. In the example under consideration $-\ \alpha = 0.20, \ \beta = 0.115$, the values are comparable in the order of magnitude not equal - they differ by $-\alpha - \beta = c_0 \ln c = 0.20 - 0.115 = 0.085$, i.e., the relationship of the threshold values approaches unity – the sensitivity of one tree species at different places of habitat is similar, and the higher are the local weights c_0 of influencing factors, the closer is the agreement.

Thus the homology of real and modelled time series is governed by the degree of concerted response of geosystem elements to climatic effects with due regard for the individual peculiar features of habitats, and for the properties of the elements. Indicator $a_j$serves as the index of this individuality (analogue of I) and determines the specific character of this response to the effect of various factors.

4. Conclusion

Since the natural and human geographical processes and phenomena have composite and have a different quality, they are not possible to model by using only one type of models. To do this requires a large number of models, each of which is based on its own systemic understanding of the modelled territorial object, an understanding that is fixed in a relevant theoretical construct.

Different systems theories are reach-through theories, i.e., they are able to describe with the same quality the physical, chemical, biological, economic, and social phenomena using the same system language having regard to the characteristic properties of the aforementioned organization levels of real processes. This allows geography to use in a modelling a universal apparatus of representation of the natural and social regularities.

A theoretical reach-through description of the reality requires new logic of research that received the name vector-combinatorial logic with the operations of negation and mediation of the opposites. It sets the rules of transition from one opposite (knowledge layer) to another, and from one system theory to another in particular. All theories for modelling from this standpoint are equivalent to within interpretation of notions, i.e., a substitution of notions in axioms from, for example, the general systems theory for notions from the special systems theory yields mathematical expressions of from the special systems theory: dynamic, functional, sophisticated and other systems. This generates a peculiar space of theoretical research into geographical and any other objects under different aspects.

The theory of complexes operating with models of systems of different qualities and their combinations is the basic theory of geographical modelling. All real processes and phenomena and their models form a complex, i.e. a linearly ordered metrized sequence of functionally connected systems, subsystems and their combinations. The model of any process or phenomenon is identified by its position in an elementary set of continuum, a segment of values [0,1]. These individual values of modelled situations are developed into coefficients of models for the particular situations and, hence, with a knowledge of the connections of the coefficients, it is possible to identify the code of the situation. One of the coefficients of the equations use can serve as such an indicator.

In modelling the geographical situation, one model combines theoretical knowledge from different theories thus generating a polymodel of a phenomenon. The results of polymodel analysis of dendroclimatic series presented in this paper allow one to distinguish different levels of the causes for the variability in tree diameter increment and to theoretically explain their origin.

With the emergence of qualitatively new problems of modelling geographical processes and phenomena, there arises a need to synthesize novel theories and respective system languages of the formal description of phenomena and processes, which is achieved through the use of methods of induction and interpretation from the general systems theory. Another commonly occurring issue is the problem of adapting existing models to particular geographical situations, which is ensured through calibration of models using identification indices. This must provide the maximum complete, comprehensive description of objects of a territory, which should be striven for through a further development of the polysystem methodology of modelling in geographical science.

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