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Preface

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Control theory has developed rapidly since the first papers by Pontryagin and collaborators in the late 1950s, and is now established as an important area of applied mathematics. Optimal control, stabilization and controllability have already found their way into many areas of modeling and control in engineering, and nowadays are strongly utilized in many other fields of applied sciences, in particular biology, medicine, economics, and finance. Research activity in control theory is seen as a source of many useful and flexible tools in decision making, such as for optimal therapies (in medicine) and strategies (in economics). The methods of control theory are drawn from a varied spectrum of mathematical results, and, on the other hand, control problems provide a rich source of deep mathematical problems. The choice of applications to either life sciences or economics takes into account modern trends of treating economic problems in osmosis with biological paradigms.

The aim of this Special Issue is to provide a tour of methods in control theory and related computational methods for ODE and PDE models and to emphasize the applications in different domains. It brings together new developments in these areas of research obtained by top specialists from ten countries, as well as illustrates applications of these results to a wide range of real-world problems.

The contents of this Special Issue reflects the recent research interests of the contributors, but we are confident that, altogether, this Special Issue represents a reasonable cross section of current mathematical research in control theory. A glance to the table of contents will convince the reader that the contributions to this Special Issue are not only of interest for those working in the control theory, but also from other fields of mathematics. Both theoretical and algorithmic developments contained in this Special Issue can be seen in the list of papers briefly presented below. The applications of the theoretical results mainly concern the following domains: biological populations, environmental sciences, medicine and economics.

The paper "Zero-Stabilization for Some Diffusive Models with State Constraints" by S. Aniţa, treats with the zero-controllability and the zero-stabilizability for the nonnegative solutions to some Fisher-like models with nonlocal terms describing the dynamics of biological populations with diffusion, logistic term and migration. A necessary and sufficient condition for the nonnegative zero-stabilizability for a linear integro-partial differential equation is obtained in terms of the sign of the principal eigenvalue to a certain non-selfadjoint operator. For a related semilinear problem a necessary condition and a sufficient condition for the locally nonnegative zero-stabilizability are also derived in terms of the magnitude of the above mentioned principal eigenvalue. The rate of stabilization corresponding to a simple feedback stabilizing control is dictated by the principal eigenvalue. A large principal eigenvalue leads to a fast stabilization to zero. A necessary condition and a sufficient condition for the stabilization to zero of the predator population in a predator-prey system is also investigated. Finally, a method to approximate the above mentioned principal eigenvalues is indicated.

The authors A.O. Belyakov and V.M. Veliov investigate in the paper "Constant versus Periodic Fishing: Age Structured Optimal Control Approach" an age-structured infinite-horizon optimal control model of harvesting a biological resource, interpreted as fish. Time and age are considered as continuum variables. The main result shows that in case of selective fishing, where only fish of prescribed sizes is harvested, it may be advantageous in the log run to implement a periodic fishing effort, rather than constant (the latter suggested by single-fish models that disregard the age-heterogeneity). Thus taking into account the age-structure of the fish may qualitatively change the theoretically optimal fishing mode. This result is obtained by developing a technique for reliable numerical verification of second order necessary optimality conditions for the considered problem. This technique could be useful for other optimal control problems of periodic age-structured systems.

The paper "Linear Size-Structured Population Models with Special Diffusion and Optimal Harvesting Problems" by N. Kato presents a linear size-structured population model with spacial diffusion for which existence of a unique mild solution is established. Then a harvesting problem for the linear size-structured models with diffusion is investigated and the existence of an optimal harvesting effort to maximize the total price or total harvest is proven.

The paper "Optimal Sustainable Policies under Pollution Ceiling: the Demographic Side" by R. Boucekkine, B. Martinez, R. Ruiz-Tamarit studies the impact of demographics dynamics on the design of optimal environmental and economic policies (sustainable policies) under the threat of catastrophic pollution. The main controls in the hands of the policymaker are pollution abatement and consumption standards (per capita). The policymaker maximizes total inter-generational welfare subject to given demographic, technological and pollution dynamics, and under a pollution ceiling level featuring the catastrophic pollution threat. The demographic and technological dynamics are exogenously given. The former are assumed logistic and calibrated in order to fit actual United Nations projections. Pollution dynamics depend on emissions, which can be controlled through abatement, and on exogenous demographic dynamics. Mathematically, the problem amounts to the optimal control of a system of nonlinear non-autonomous differential equations subject to a pure state constraint (pollution ceiling). Using a heuristic method to indentify asymptotic dynamics, a sufficient condition for catastrophic pollution is identified. When the economy goes to the pollution ceiling value, it stays on, involving a more stringent environmental policy and a sacrifice in terms of consumption per capita. The optimal policies are altered when it is departed from the logistic world by considering exponential technical progress (keeping population growth logistic). It is shown that introducing such an asymmetry widens the margins of optimal policies as sustainable environmental policies are clearly less stringent under exponential technical progress. Finally, the model is connected to the data using, in particular, UN population projections.

One of relevant issues of the modern climate change studies is to combine two major strategies to deal with climate change: mitigation of greenhouse gases emissions and adaptation to global warming. The paper "Modeling of Environmental Adaptation Versus Pollution Mitigation" of Y. Yatsenko, N. Hritonenko, and T. Bréchet contributes to this emerging issue. The paper analyzes a nonlinear optimal control problem that describes how environmental pollution, mitigation, and adaptation impact the long-run economic-environmental development. Provided steady-state analysis of the problem reveals important economic implications of the obtained qualitative dynamics. Analytic results are supplemented with numeric simulation. The paper demonstrates that the capital deterioration cannot be ignored in designing optimal policies to combat climate change and that the optimal policy mix of adaptation and abatement depends on the country's economic potential. In the case of a poor country, the optimal environmental policy may be no adaptation to climate change at all. The maximum adaptation efforts (in terms of the adaptation/abatement ratio) should be done by countries at intermediate stage of development.

The study “Evolutionary Dynamics and Optimal Control of Chemotherapy in Cancer Cell Populations under Immune Selection Pressure” (authors: G. Dimitriu, T. Lorenzi, and R. Ştefănescu) presents a mathematical model for the dynamics of cancer and immune cells under the effects of chemotherapy and immunity-boosters. Tumor cells are modeled as a population structured by a continuous phenotypic trait, that is related to the level of resistance to receptor-induced cell death triggered by effector lymphocytes. In an optimal control framework, the authors tackle the problem of designing effective anti-cancer protocols. The results obtained suggest that chemotherapeutic drugs characterized by high cytotoxic effects can be useful for treating tumors of large size. On the other hand, less cytotoxic chemotherapy in combination with immunity-boosters can be effective against tumors of smaller size. Taken together, these results support the development of therapeutic protocols relying on combinations of less cytotoxic agents and immune-boosters to fight cancer in the early stages.

The paper “Optimal Vaccination, Treatment, and Preventive Campaigns in Regard to the SIR Epidemic Model” presented by E.V. Grigorieva and E.N. Khailov considers the optimal control of the Susceptible-Infected-Recovered (SIR) model for the spread of an infectious disease in a population of constant size. In order to control the spread of infection, the SIR model with four bounded controls (vaccination of newborns, vaccination of the susceptible, treatment of infected, and indirect strategies aimed at a reduction of the incidence rate) is suggested and investigated. The authors state the optimal control problem of minimizing the total number of infected individuals on a given time interval and find the optimal solutions analytically through qualitative investigation of the switching functions. Analytical investigation of the model reduces a complex, two-point boundary value problem of the Maximum Principle to the much simpler problem of one-dimensional constrained optimization. It is demonstrated how parameters of the model influence the choice of the optimal policy. This work is unique in the perspective that multiple controls are mathematically analyzed jointly. When four controls are applied simultaneously, it is found that for some model parameters, one of the control functions can have a piecewise constant form, which implies that the implementation of precocious methods (quarantine, wearing masks and closing public events) can be delayed until a later time. A detailed description of the approach employed for solving the considered problem as well as the results of the numerical calculations and their discussion are also discussed.

The authors U. Ledzewicz and H. Schättler present in the paper “A Review of Optimal Protocols: From MTD towards Metronomic Therapy” mathematical results concerning the qualitative structure of chemotherapy protocols that were obtained with the methods of optimal control. As increasingly more complex features are incorporated into the mathematical model (progressing from models for homogeneous, chemotherapeutically sensitive tumor cell populations to models for heterogeneous agglomerations of subpopulations of various sensitivities to models that include tumor immune-system interactions), the structures of optimal controls change from bang-bang solutions (which correspond to maximum dose rate chemotherapy with restperiods) to solutions that favor singular controls. Medically, this corresponds to a transition from standard maximum tolerated dose (MTD) type protocols to chemo-switch strategies towards metronomic dosing.

The paper “Optimal Control Applied in Coupled Within-Host and Between-Host Models” (authors: E. Numfor, S. Bhattacharya, S. Lenhart, and M. Martcheva) formulates an immuno-epidemiological model of coupled “within-host” model of ODEs and “between-host” model of ODE and PDE, using the Human Immunodeficiency Virus (HIV) for illustration. Existence and uniqueness of solution to the “between-host” model is established, and an explicit expression for the basic reproduction number of the “between-host” model is derived. Stability of disease-free and endemic equilibria is also investigated. An optimal control problem with drug-treatment control on the within-host system is formulated and analyzed. Numerical simulations based on the forward-backward sweep method are obtained.

The study “Optimal Protocols for the Anti-VEGF Tumor Treatment” (authors: J. Poleszczuk, M.J. Piotrowska, U. Forys) concerns well established family of angiogenesis models together with their recently proposed modification, that increases accuracy in the case of treatment using VEGF anthibodies. The authors consider the optimal control problem of minimizing the tumor volume when the maximal admissible drug dose and the final level of vascularization are taken into account. They show that the optimal strategy consists of drug-free, full-dose and singular (with intermediate values of the control variable) intervals. Moreover, no bang-bang switch of the control is possible. Numerical results with Matlab and based on gradient and shooting methods are also given to illustrate the theoretical results.

A. Swierniak and J. Klamka investigate in their paper "Local Controllability of Models of Combined Anticancer Therapy with Delays in Control" the local controllability of models of combined anticancer therapy, the combination of antiangiogenic therapy with conventional treatment being considered one of the most inspiring approaches in modern oncology. From a mathematical point of view, the influence of more than one therapy and not only one kind of drug becomes a multi-control problem. In order to include in the model the pharmacokinetic-pharmacodynamic effects of antiangiogenic and cytostatic agents, the authors propose to introduce delays in control variables. Another reason for delays in chemotherapy dosing is related to an idea that angiogenic therapy should be implemented first considering that the vascular network should be normalized before chemotherapy. The authors consider relative constrained local controllability for second-order finite-dimensional semilinear stationary dynamical systems described by the set of two ordinary differential state equations with delays.

The paper "Control Approach to an Ill-Posed Variational Inequality" (author: G. Marinoschi) takes into discussion a boundary value problem for a parabolic equation which contains the subdifferential of the indicator function of a closed convex set on the right-hand side of the equation. In fact, the equation turns out to be an ill-posed variational inequality. Such a model may arise from absorption-desorption processes, as for example oxygen absorbed by biological tissues, fluids absorbed into the pores of a porous medium, or heat retained or ceded by materials. This ill-posed variational inequality is approached as an optimal control problem, being translated into a constrained minimization problem with a nonconvex functional involving the inverse graph of the indicator function subdifferential. The existence of a variational or generalized solution to the original inclusion, defined as a solution to the optimal control problem is proved. By using the optimality conditions, a computing algorithm for calculating the variational solution is presented and results of some numerical experiments for an one-dimensional problem are given.

The paper "A Numerical Method with Singular Perturbation to Approximate the Controls of the Heat Equation" (authors: I.F. Bugariu and S. Micu) is devoted to the analysis of a numerical scheme for the approximation of the linear heat equation's controls. Due to the regularizing effect, the efficient computation of the null controls for parabolic type equations is a difficult problem. A possible cure for the bad numerical behavior of the approximating controls consists of adding a singular perturbation depending on a small parameter ε which transforms the heat equation into a wave equation. A space discretization of step h leads to a system of ordinary differential equations. The aim of the paper is to prove that there exists a sequence of exact controls of the corresponding perturbed semi-discrete systems which converges to a control of the original heat equation when both h (the mesh size) and ε (the perturbation parameter) tend to zero.

We hope that the scope of this Special Issue makes it both a good reference to researchers active in the area as well as a convenient source to introduce the general mathematical reader to this field which brings together many branches of mathematics.

We wish to express our sincere appreciation to all those who have contributed to the completion of this Special Issue. In particular, we are deeply grateful to our referees who provided prompt and extensive reviews for all the submitted papers. The number and high quality of the contributions demonstrates the vitality and creativity of optimal control theory and its applications and shows the continuing need for special issues of this kind. We also wish to thank the Editor-in-Chief of the journal "Mathematical Modelling and Natural Phenomena", Professor Vitali Volpert, for his kind cooperation and professional support.

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