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A COMPUTATIONAL STUDY OF TRANSMISSION DYNAMICS FOR DENGUE FEVER WITH A FRACTIONAL APPROACH

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Abstract. Fractional derivatives are considered an influential weapon in terms of analysis of infectious diseases because of their nonlocal nature. The inclusion of the memory effect is the prime advantage of fractional-order derivatives. The main objective of this article is to investigate the transmission dynamics of dengue fever, we consider generalized Caputo-type fractional derivative (GCFD) (${}^{\mathrm{C}}\mathrm{D}_0^{\beta,\sigma}$) for alternate representation of dengue fever disease model. We discuss the existence and uniqueness of the solution of model by using fixed point theory. Further, an adaptive predictor-corrector technique is utilized to evaluate the considered model numerically.

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1. Introduction

Bacteria and viruses are the sources of many infectious diseases that are very dangerous for human health. In infectious disease, dengue fever is one of the extremely serious diseases which is caused by a virus spread by mosquitoes, threatening about 2.5 billion people specifically in tropical countries. Southeast Asia is the most affected region by epidemic dengue fever [37]. This epidemic has seasonal patterns which often seen during and after monsoon seasons and this can also explain climate change. So, there is a need for a mathematical model to knowledge about the awareness of dengue fever. In the last few decades, increasingly study on mathematical models including dengue fever disease (DFD) were found [8, 9, 11, 13, 21, 33, 34, 36].

The investigation of biological models is now becoming a growing area of research, the scientists and researchers across the globe giving more importance to these models due to their impact on human life. The biological mathematical models provide a better idea about the disease transmission and impact of disease in the community. Many studies have been done on the stability theory and the existence and uniqueness results of biological models [3, 14, 20, 23, 35, 40].

Keywords and phrases: Fractional derivative, generalized Caputo derivative, dengue fever model, existence and uniqueness, adaptive P-C technique.

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In this article, we consider a Susceptible-Infected-Recovered (SIR) based DFD model [37]. The integer-order model represented by three sets of equations is given as:

$$\begin{aligned}
&\dot{\mathrm{S}}(t) = \nu - (\nu + \alpha\mathrm{R})\mathrm{S},\\
&\dot{\mathrm{E}}(t) = \alpha\mathrm{SR} - \gamma\mathrm{E},\\
&\dot{\mathrm{R}}(t) = \lambda\mathrm{E} - (\lambda\mathrm{E} + \delta)\mathrm{R},
\end{aligned} \tag{1.1}$$

where S is the population of susceptible people, E represents the infected people with dengue virus and R represents the number of recovered people from dengue virus. The parameters used in model have their own meaning given as follows: the parameter ν denote the death rate of susceptible people, α denotes the average number of bites per infected mosquito, γ represents the rate of infection, the rate of recovery after the infection is represented by λ and the number of deaths among the susceptible mosquito is represented by δ.

The work in this article is based on alternate description of model in frame of GCFD which is given as:

$$\begin{aligned}
&{}^{\mathrm{C}}\mathrm{D}_0^{\beta,\sigma}\mathrm{S}(t) = \nu - (\nu + \alpha\mathrm{R})\mathrm{S},\\
&{}^{\mathrm{C}}\mathrm{D}_0^{\beta,\sigma}\mathrm{E}(t) = \alpha\mathrm{SR} - \gamma\mathrm{E},\\
&{}^{\mathrm{C}}\mathrm{D}_0^{\beta,\sigma}\mathrm{R}(t) = \lambda\mathrm{E} - (\lambda\mathrm{E} + \delta)\mathrm{R},
\end{aligned} \tag{1.2}$$

with $\mathrm{S}(0) = \mathrm{N}_1, \mathrm{E}(0) = \mathrm{N}_2$, and $\mathrm{R}(0) = \mathrm{N}_3$ as initial conditions, where $0 < \beta \leq 1$ and $\sigma > 0$. The total population is denoted by N_p and separated into three subclasses susceptible, infected and recovered people given as $\mathrm{N}_p = \mathrm{S}(t) + \mathrm{E}(t) + \mathrm{R}(t)$.

Fractional calculus (FC) is rising as popular field of research in applied mathematics. In recent years, the development of models with variable order derivatives from different fields is increasing rapidly. The reason behind this is the competency of FC to capture memory and the hereditary nature of real-world problems. For application range of FC we refer [2, 5, 15, 16, 24–28, 38, 39]. The non-local nature of fractional derivatives gives them greater advantages over integer order. In literature different form of fractional derivative are exist in them the Caputo derivative [7] is most famous and largely used by researchers. Recently, a generalized version of fractional integral and derivatives are presented in [19, 30], this generalized Caputo-type derivative has been used in investigation of many physical models [1, 4, 6, 12, 17, 31].

The rest of the article is arranged into many sections, which start with some new novel definitions of fractional derivative given in Section 2. Existence and uniqueness for the fractional model are discussed *via* fixed point theory, in Section 3. A new numerical scheme and its application to considered model is given in Section 4. In Section 5, simulations are presented graphically. Lastly, the outcomes are given in Section 6.

2. Primary preliminaries

In this section, first we briefly review some novel definition fractional integral and derivative [22, 29, 32].

Definition 2.1. The Riemann-Liouville fractional integral of a function $\Phi(t)$ and order β is defined as:

$${}_0\mathrm{I}^{\beta}(\Phi(t)) = \frac{1}{\Gamma(\beta)}\int_0^t (t-\eta)^{\beta-1}\Phi(\eta)\mathrm{d}\eta, \tag{2.1}$$

where $\Gamma(.)$ is known as gamma function.

Definition 2.2. Let the function $\Phi(t)$ be real and differentiable function of order $\beta \in [0,1)$, then Caputo derivative is defined as:

$${}_0^{\mathrm{C}}\mathrm{D}^{\beta}\Phi(t) = \mathrm{I}^{m-\beta}\mathrm{D}^m\Phi(t) = \frac{1}{\Gamma(m-\beta)}\int_0^t (t-\eta)^{m-\beta-1}\Phi^m(\eta)\mathrm{d}\eta. \tag{2.2}$$

Now, we briefly present some key definitions of generalized fractional operators that we will use in this work, given in [18, 19, 30].

Definition 2.3. For $\beta \in (m-1, m]$, where m belongs to natural number, the generalized fractional integral $\mathrm{I}_{a^+}^{\beta,\sigma}$, of a function $\Phi(t)$, is defined by (if integral exist)

$$\mathrm{I}_{a^+}^{\beta,\sigma}(\Phi(t)) = \frac{\sigma^{1-\beta}}{\Gamma(\beta)} \int_a^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1}\Phi(\eta)\mathrm{d}\eta, \quad t > a, \tag{2.3}$$

where $\sigma > 0$ and $a \geq 0$.

Definition 2.4. For $\beta \in (m-1, m]$, where m belongs to natural number and $\sigma > 0$, $a \geq 0$, the generalized Caputo-type fractional derivative of variable order β of a function $\Phi(t)$ is defined as:

$${}^{\mathrm{C}}\mathrm{D}_{a^+}^{\beta,\sigma}\Phi(t) = \frac{\sigma^{\beta-m+1}}{\Gamma(m-\beta)} \int_a^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{m-\beta-1}\big(\eta^{1-\sigma}\frac{\mathrm{d}}{\mathrm{d}\eta}\big)^m\Phi(\eta)\mathrm{d}\eta, \quad t > a. \tag{2.4}$$

3. Existence and uniqueness of solution

Here, we will examine the existence and uniqueness of solution of the fractional model (1.2), by powerful tool fixed-point theory. Now, we consider DFD system in view of definition (2.3), we obtain

$$\begin{aligned} \mathrm{S}(t) - \mathrm{S}(0) = &\quad \mathrm{I}_{0^+}^{\beta,\sigma}\{\nu - [\nu + \alpha\mathrm{R}]\mathrm{S}\}, \\ \mathrm{E}(t) - \mathrm{E}(0) = &\quad \mathrm{I}_{0^+}^{\beta,\sigma}\{\alpha\mathrm{SR} - \gamma\mathrm{E}\}, \\ \mathrm{R}(t) - \mathrm{R}(0) = &\quad \mathrm{I}_{0^+}^{\beta,\sigma}\{\lambda\mathrm{E} - [\lambda\mathrm{E} + \delta]\mathrm{R}\}. \end{aligned} \tag{3.1}$$

For simplicity we consider following kernel

$$\begin{aligned} \mathrm{V}_1(t, \mathrm{S}) = &\quad \nu - [\nu + \alpha\mathrm{R}]\mathrm{S}, \\ \mathrm{V}_2(t, \mathrm{E}) = &\quad \alpha\mathrm{SR} - \gamma\mathrm{E}, \\ \mathrm{V}_3(t, \mathrm{R}) = &\quad \lambda\mathrm{E} - [\lambda\mathrm{E} + \delta]\mathrm{R}. \end{aligned} \tag{3.2}$$

Thus

$$\begin{aligned} \mathrm{S}(t) - \mathrm{S}(0) &= \tfrac{\sigma^{1-\beta}}{\Gamma(\beta)} \textstyle\int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1}\mathrm{V}_1(\eta, S)\mathrm{d}\eta, \\ \mathrm{E}(t) - \mathrm{E}(0) &= \tfrac{\sigma^{1-\beta}}{\Gamma(\beta)} \textstyle\int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1}\mathrm{V}_2(\eta, E)\mathrm{d}\eta, \\ \mathrm{R}(t) - \mathrm{R}(0) &= \tfrac{\sigma^{1-\beta}}{\Gamma(\beta)} \textstyle\int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1}\mathrm{V}_3(\eta, R)\mathrm{d}\eta. \end{aligned} \tag{3.3}$$

Theorem 3.1. *The kernels* $\mathrm{V}_1, \mathrm{V}_2$ *and* V_3 *satisfy the Lipschitz condition and contraction, if holds the following inequality*

$$0 \leq \Omega_1, \Omega_2, \Omega_3 < 1.$$

Proof. Let S and S_1 are the two functions for the kernel V_1, then we have

$$\|\mathrm{V}_1(t, S) - \mathrm{V}_1(t, S_1)\| = \| -(\nu + \alpha\mathrm{R})(\mathrm{S} - \mathrm{S}_1)\|, \tag{3.4}$$

by property of norm, we have

$$\begin{aligned} \|\mathrm{V}_1(t,S)-\mathrm{V}_1(t,S_1)\| &\leq \big(\nu+\alpha\|\mathrm{R}\|\big)\|\mathrm{S}-\mathrm{S}_1\|,\\ &\leq \big(\nu+\alpha d_3\big)\|\mathrm{S}-\mathrm{S}_1\|,\\ &\leq \Omega_1\|\mathrm{S}-\mathrm{S}_1\|. \end{aligned} \tag{3.5}$$

We assume $\Omega_1=\nu+\alpha d_3<1$. Here, we note that $\|\mathrm{S}\|\leq d_1, \|\mathrm{E}\|\leq d_2$ and $\|\mathrm{R}\|\leq d_3$ is the bounded function. Thus, we have

$$\|\mathrm{V}_1(t,S)-\mathrm{V}_1(t,S_1)\|\leq\Omega_1\|\mathrm{S}-\mathrm{S}_1\|. \tag{3.6}$$

Hence, the kernel V_1 satisfy the Lipschitz condition, and if $0\leq\Omega<1$ then it is contraction for V_1. Similarly, the following inequality for remaining kernels can be obtained

$$\begin{aligned} &\|\mathrm{V}_2(t,\mathrm{E})-\mathrm{V}_2(t,\mathrm{E}_1)\|\leq\Omega_2\|\mathrm{E}-\mathrm{E}_1\|,\\ &\|\mathrm{V}_3(t,\mathrm{R})-\mathrm{V}_3(t,\mathrm{R}_1)\|\leq\Omega_3\|\mathrm{R}-\mathrm{R}_1\|. \end{aligned} \tag{3.7}$$

Now, we give the recursive formula by using equation (3.3), which is given as

$$\begin{aligned} \mathrm{S}_n(t)=&\quad \mathrm{S}(0)+\tfrac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\mathrm{V}_1(\eta,\mathrm{S}_{n-1})\mathrm{d}\eta,\\ \mathrm{E}_n(t)=&\quad \mathrm{E}(0)+\tfrac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\mathrm{V}_2(\eta,\mathrm{E}_{n-1})\mathrm{d}\eta,\\ \mathrm{R}_n(t)=&\quad \mathrm{R}(0)+\tfrac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\mathrm{V}_3(\eta,\mathrm{R}_{n-1})\mathrm{d}\eta, \end{aligned} \tag{3.8}$$

with the initial conditions

$$\mathrm{S}(t_0)=\mathrm{S}(0),\ \mathrm{E}(t_0)=\mathrm{E}(0),\ \mathrm{R}(t_0)=\mathrm{R}(0).$$

Through difference between successive terms, we have following expression

$$\begin{aligned} \theta_n(t)=&\quad \mathrm{S}_n(t)-\mathrm{S}_{n-1}(t)=\tfrac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\big(\mathrm{V}_1(\eta,\mathrm{S}_{n-1})-\mathrm{V}_1(\eta,\mathrm{S}_{n-2})\big)\mathrm{d}\eta,\\ \phi_n(t)=&\quad \mathrm{E}_n(t)-\mathrm{E}_{n-1}(t)=\tfrac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\big(\mathrm{V}_2(\eta,\mathrm{E}_{n-1})-\mathrm{V}_2(\eta,\mathrm{E}_{n-2})\big)\mathrm{d}\eta,\\ \psi_n(t)=&\quad \mathrm{R}_n(t)-\mathrm{R}_{n-1}(t)=\tfrac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\big(\mathrm{V}_3(\eta,\mathrm{R}_{n-1})-\mathrm{V}_3(\eta,\mathrm{R}_{n-2})\big)\mathrm{d}\eta. \end{aligned} \tag{3.9}$$

It is clear that

$$\mathrm{S}_n(t)=\sum_{i=1}^n\theta_i(t),\qquad \mathrm{E}_n(t)=\sum_{i=1}^n\phi_i(t),\qquad \mathrm{R}_n(t)=\sum_{i=1}^n\psi_i(t).$$

Performing the norm on equation (3.9) and using the triangular inequality, we get

$$\begin{aligned} \|\theta_n(t)\|=&\|\mathrm{S}_n(t)-\mathrm{S}_{n-1}(t)\|\\ &\leq\frac{\sigma^{1-\beta}}{\Gamma(\beta)}\left\|\int_0^t\eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\big(\mathrm{V}_1(\eta,\mathrm{S}_{n-1})-\mathrm{V}_1(\eta,\mathrm{S}_{n-2})\big)\mathrm{d}\eta\right\|. \end{aligned} \tag{3.10}$$

As earlier we have shown that the kernel satisfies Lipschitz condition, we obtain

$$\|\mathrm{S}_n(t) - \mathrm{S}_{n-1}(t)\| \leq \frac{\Omega_1 \sigma^{1-\beta}}{\Gamma(\beta)} \int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} \|\mathrm{S}_{n-1} - \mathrm{S}_{n-2}\| \mathrm{d}\eta. \tag{3.11}$$

Thus we have

$$\|\theta_n(t)\| \leq \frac{\Omega_1 \sigma^{1-\beta}}{\Gamma(\beta)} \int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} \|\theta_{n-1}(\eta)\| \mathrm{d}\eta. \tag{3.12}$$

Similarly, we can obtain the expression for

$$\begin{aligned} \|\phi_n(t)\| &\leq \tfrac{\Omega_2 \sigma^{1-\beta}}{\Gamma(\beta)} \int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} \|\phi_{n-1}(\eta)\| \mathrm{d}\eta, \\ \|\psi_n(t)\| &\leq \tfrac{\Omega_3 \sigma^{1-\beta}}{\Gamma(\beta)} \int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} \|\psi_{n-1}(\eta)\| \mathrm{d}\eta. \end{aligned} \tag{3.13}$$

Theorem 3.2. *The solution of the system exist, if there exist $t^\sigma_{\max}$ such that*

$$\left(\frac{t^\sigma_{\max}}{\sigma}\right)^\beta \frac{\Omega_1}{\Gamma(1+\beta)} < 1.$$

Proof. As we know that the $\mathrm{S}(t), \mathrm{E}(t), \mathrm{R}(t)$ are the bounded functions and kernels hold the Lipschitz condition. The following succeeding relations can be obtained using equations (3.12) and (3.13) given as:

$$\begin{aligned} \|\theta_n(t)\| &\leq \|\mathrm{S}_0\| \left[\left(\tfrac{t^\sigma_{\max}}{\sigma}\right)^\beta \tfrac{\Omega_1}{\Gamma(1+\beta)}\right]^n, \\ \|\phi_n(t)\| &\leq \|\mathrm{E}_0\| \left[\left(\tfrac{t^\sigma_{\max}}{\sigma}\right)^\beta \tfrac{\Omega_2}{\Gamma(1+\beta)}\right]^n, \\ \|\psi_n(t)\| &\leq \|\mathrm{R}_0\| \left[\left(\tfrac{t^\sigma_{\max}}{\sigma}\right)^\beta \tfrac{\Omega_3}{\Gamma(1+\beta)}\right]^n. \end{aligned} \tag{3.14}$$

Therefore, the solution of the model exists and is smooth. To prove that the above functions are the solutions of the system, we consider that

$$\begin{aligned} \mathrm{S}(t) - \mathrm{S}(0) &= \mathrm{S}_n(t) - \mathbb{C}_n(t), \\ \mathrm{E}(t) - \mathrm{E}(0) &= \mathrm{E}_n(t) - \mathbb{F}_n(t), \\ \mathrm{R}(t) - \mathrm{R}(0) &= \mathrm{R}_n(t) - \mathbb{Z}_n(t). \end{aligned} \tag{3.15}$$

Here, the aim is to show that when $n \to \infty$ the term $\|\mathbb{C}_n(t)\|$ goes to zero. Now, considering the norm and Lipschitz condition for the kernel V_1, we get

$$\begin{aligned} \|\mathbb{C}_n(t)\| &\leq \left\| \tfrac{\sigma^{1-\beta}}{\Gamma(\beta)} \int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} \big(\mathrm{V}_1(\eta, \mathrm{S}) - \mathrm{V}_1(\eta, \mathrm{S}_{n-1})\big) \mathrm{d}\eta \right\|. \\ &\leq \tfrac{\sigma^{1-\beta}}{\Gamma(\beta)} \int_0^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} \left\| \mathrm{V}_1(\eta, \mathrm{S}) - \mathrm{V}_1(\eta, \mathrm{S}_{n-1}) \right\| \mathrm{d}\eta. \\ &\leq \left(\tfrac{t^\sigma}{\sigma}\right)^\beta \tfrac{\Omega_1}{\Gamma(1+\beta)} \|\mathrm{S} - \mathrm{S}_{n-1}\|. \end{aligned} \tag{3.16}$$

Repeating the same process, we obtain

$$\|\mathbb{C}_n(t)\| \leq \|\mathrm{S}(0)\|\Big[\Big(\frac{t^\sigma}{\sigma}\Big)^\beta \frac{1}{\Gamma(1+\beta)}\Big]^{n+1}\Omega_1^n\mathrm{M}, \tag{3.17}$$

at $t_{\max}$, we have

$$\|\mathbb{C}_n(t)\| \leq \|\mathrm{S}(0)\|\Big[\Big(\frac{t_{\max}^\sigma}{\sigma}\Big)^\beta \frac{1}{\Gamma(1+\beta)}\Big]^{n+1}\Omega_1^n\mathrm{M}. \tag{3.18}$$

Apply the limit on equation (3.18), when limit $n \to \infty$, we obtain $\|\mathbb{C}_n(t)\| \to 0$. Similarly, we can establish $\|\mathbb{F}_n(t)\| \to 0$, and $\|\mathbb{Z}_n(t)\| \to 0$.

Another significant aspect is to establish the uniqueness of solutions of the model. To show the solution is unique, we assume that $\mathrm{S}_1(t), \mathrm{E}_1(t)$ and $\mathrm{R}_1(t)$, be another solutions of the model (1.2), then

$$\mathrm{S}(t) - \mathrm{S}_1(t) = \frac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t \eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\big(\mathrm{V}_1(\eta,\mathrm{S}) - \mathrm{V}_1(\eta,\mathrm{S}_1)\big)\mathrm{d}\eta. \tag{3.19}$$

Using norm on equation (3.19), we get

$$\|\mathrm{S}(t) - \mathrm{S}_1(t)\| \leq \frac{\sigma^{1-\beta}}{\Gamma(\beta)}\int_0^t \eta^{\sigma-1}(t^\sigma-\eta^\sigma)^{\beta-1}\|\mathrm{V}_1(\eta,\mathrm{S}) - \mathrm{V}_1(\eta,\mathrm{S}_1)\|\mathrm{d}\eta, \tag{3.20}$$

because the kernel satisfies the Lipschitz condition, we get

$$\|\mathrm{S}(t) - \mathrm{S}_1(t)\| \leq \Big(\frac{t^\sigma}{\sigma}\Big)^\beta \frac{\Omega_1}{\Gamma(1+\beta)}\|\mathrm{S}(t) - \mathrm{S}_1(t)\|, \tag{3.21}$$

$$\|\mathrm{S}(t) - \mathrm{S}_1(t)\|\Big[1 - \Big(\frac{t^\sigma}{\sigma}\Big)^\beta \frac{\Omega_1}{\Gamma(1+\beta)}\Big] \leq 0, \tag{3.22}$$

$$\|\mathrm{S}(t) - \mathrm{S}_1(t)\| = 0, \Longrightarrow \mathrm{S}(t) = \mathrm{S}_1(t). \tag{3.23}$$

Thus, we achieved the uniqueness of the system solution. In similar manner, we can prove that

$$\mathrm{E}(t) = \mathrm{E}_1(t), \qquad \mathrm{R}(t) = \mathrm{R}_1(t). \tag{3.24}$$

□

4. Numerical technique

Numerical techniques are taken for solutions when all the existing analytical technique are fails. Numerical schemes are considered as best tool for biological model solutions modelled with fractional operators. Here, we describe the new adaptive predictor-corrector (P-C) method proposed in [30], this method is extension of predictor-corrector method proposed in [10]. Let's consider an initial value problem of the form

$$\begin{cases} \mathrm{D}_{a^+}^{\beta;\sigma}\Phi(t) = f(t,\Phi(t)), & t \in [0,T], \\ \Phi^n(a) = \Phi_0^n, & n = 0,1,\ldots,\lceil\beta\rceil, \end{cases} \tag{4.1}$$

where $\beta \in (m-1, m]$, $a \geq 0$ and $\sigma > 0$ and $\Phi \in C^m([a, \mathrm{T}])$. The equation (4.1) is equivalent, using ([30], Thm. 3) to Volterra integral equation

$$\Phi(t) = v(t) + \frac{\sigma^{1-\beta}}{\Gamma(\beta)} \int_a^t \eta^{\sigma-1}(t^\sigma - \eta^\sigma)^{\beta-1} f(\eta, \Phi(\eta))\mathrm{d}\eta, \tag{4.2}$$

where, $v(t) = \sum_{k=0}^{m-1} \frac{1}{\sigma^k k!}(t^\sigma - a^\sigma)^k \big[\big(y^{1-\sigma}\frac{\mathrm{d}}{\mathrm{d}y}\big)^k \Phi(y)\big]\big|_{y=a}$.

The first step of our method, according to the hypothesis that the function Φ such that a unique solution exists on some interval [a, T], the interval [a, T] divided into N unequal subinterval $\{[t_n, t_{n+1}],\ n = 0, 1, \ldots, \mathrm{N}-1\}$, using mesh points

$$\begin{cases} t_0 &= a, \\ t_{n+1} &= (t_n^\sigma + \hbar)^{\frac{1}{\sigma}},\ \ n = 0, 1, \ldots, \mathrm{N}-1, \end{cases} \tag{4.3}$$

where $\hbar = \frac{(\mathrm{T}^\sigma - a^\sigma)}{\mathrm{N}}$ and N is a natural number. Now, we are going to develop the approximations Φ_n, $n = 0, 1, 2, \ldots, N$, to solve equation (4.1). The basic step, assuming that we have already calculated the approximations $\Phi_\ell \approx \Phi(t_\ell), \ell = 1, 2, \ldots, n$, is that we want to get the approximate solution $\Phi_{n+1} \approx \Phi(t_n + 1)$ with the integral equation

$$\Phi(t_{n+1}) = v(t_{n+1}) + \frac{\sigma^{1-\beta}}{\Gamma(\beta)} \int_a^{t_{n+1}} \eta^{\sigma-1}(t_{n+1}^\sigma - \eta^\sigma)^{\beta-1} f(\eta, \Phi(\eta))\mathrm{d}\eta. \tag{4.4}$$

Making the substitution

$$\mu = \eta^\sigma, \tag{4.5}$$

we get,

$$\Phi(t_{n+1}) = v(t_{n+1}) + \frac{\sigma^{-\beta}}{\Gamma(\beta)} \int_{a^\sigma}^{t_{n+1}^\sigma} (t_{n+1}^\sigma - \mu)^{\beta-1} f(\mu^{1/\sigma}, \Phi(\mu^{1/\sigma}))\mathrm{d}\mu, \tag{4.6}$$

then

$$\Phi(t_{n+1}) = v(t_{n+1}) + \frac{\sigma^{-\beta}}{\Gamma(\beta)} \sum_{\ell=0}^{n} \int_{t_\ell^\sigma}^{t_{\ell+1}^\sigma} (t_{n+1}^\sigma - \mu)^{\beta-1} f(\mu^{1/\sigma}, \Phi(\mu^{1/\sigma}))\mathrm{d}\mu. \tag{4.7}$$

Next, if we use the trapezoidal quadrature formula with respect to the weight function $(t_{n+1}^\sigma - .)^{\beta-1}$ to approximate the integrals on the right-hand side of equation (4.7) replacing the function $f(\mu^{1/\sigma}, \Phi^{1/\sigma})$ by its piecewise linear interpolant with nodes chosen at the t_ℓ^σ ($\ell = 0, 1, \ldots, n+1$), then we get

$$\begin{aligned} &\int_{t_\ell^\sigma}^{t_{\ell+1}^\sigma} (t_{n+1}^\sigma - \mu)^{\beta-1} f(\mu^{1/\sigma}, \Phi(\mu^{1/\sigma}))\mathrm{d}\mu \\ &\approx \frac{\hbar^\beta}{\beta(\beta+1)} \Big\{ \big((n-\ell)^{\beta+1} - (n-\ell-\beta)(n-\ell+1)^\beta\big) f(t_\ell, \Phi(t_\ell)) \\ &+ \big((n-\ell+1)^{\beta+1} - (n-\ell+\beta+1)(n-\ell)^\beta\big) f(t_{\ell+1}, \Phi(t_{\ell+1})) \Big\}, \end{aligned} \tag{4.8}$$

with the help of equations (4.7) and (4.8), we obtain the corrector formula for $\Phi(t_{n+1})$, $\{n = 0, 1, \ldots, \mathrm{N}-1\}$,

$$\Phi(t_{n+1}) \approx v(t_{n+1}) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \sum_{\ell=0}^{n} \Delta_{\ell,n+1} f(t_\ell, \Phi(t_\ell)) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} f(t_{n+1}, \Phi(t_{n+1})), \tag{4.9}$$

where the weights $\Delta_{\ell,n+1}$ is defined as:

$$\Delta_{\ell,n+1} = \begin{cases} n^{\beta+1} - (n-\beta)(n+1)^{\beta}, & if\; \ell = 0, \\ (n-\ell+2)^{\beta+1} + (n-\ell)^{\beta+1} - 2(n-\ell+1)^{\beta+1}, & if\; 1 \le \ell \le n. \end{cases} \tag{4.10}$$

The final step of our method is to replace the quantity $\Phi(t_{n+1})$ present on the right side of the equation (4.9) with the quantity $\Phi^{\mathrm{P}}(t_{n+1})$ is called predictor value which can be obtained by using one-step Adams-Bashforth method to the equation (4.6). For this, the function $f(\mu^{1/\sigma}, \Phi(\mu^{1/\sigma}))$ is replaced by $f(t_j, \Phi(t_j))$ in equation (4.7) at each integral, we get

$$\begin{aligned} \Phi^{\mathrm{P}}(t_{n+1}) &\approx v(t_{n+1}) + \frac{\sigma^{-\beta}}{\Gamma(\beta)} \sum_{\ell=0}^{n} \int_{t_\ell^\sigma}^{t_{\ell+1}^\sigma} (t_{n+1}^\sigma - \mu)^{\beta-1} f(t_\ell, \Phi(t_\ell)) \mathrm{d}\mu, \\ &= v(t_{n+1}) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+1)} \sum_{\ell=0}^{n} [(n+1-\ell)^{\beta} - (n-\ell)^{\beta}] f(t_\ell, \Phi(t_\ell)). \end{aligned} \tag{4.11}$$

Therefore, the adaptive P-C technique, for calculating the approximation $\Phi_{n+1} \approx \Phi(t_{n+1})$ is fully described by the formula

$$\Phi_{n+1} \approx v(t_{n+1}) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \sum_{\ell=0}^{n} \Delta_{\ell,n+1} f(t_\ell, \Phi_\ell) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} f(t_{n+1}, \Phi_{n+1}^{\mathrm{P}}), \tag{4.12}$$

where $\Phi_\ell \approx \Phi(t_\ell)$, $\ell = 0, 1, \ldots, n$, and the predicted value $\Phi_{n+1}^{\mathrm{P}} \approx \Phi^{\mathrm{P}}(t_{n+1})$ can be determined as expressed in equation (4.11). When $\sigma = 1$, the adaptive P-C technique is reduced to P-C technique given in [10].

Now, we apply the above adaptive P-C approach on DFD model (1.2). Taking the iterative solution form given in equation (4.12), approximations $\mathrm{S}_{n+1}, \mathrm{E}_{n+1}$ and R_{n+1} are defined as:

$$\begin{aligned} \mathrm{S}_{n+1} \approx \mathrm{S}(0) + &\frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \sum_{\ell=0}^{n} \Delta_{\ell,n+1} \mathcal{G}_1(t_\ell, \mathrm{S}_\ell, \mathrm{E}_\ell, \mathrm{R}_\ell) \\ &+ \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \mathcal{G}_1(t_{n+1}, \mathrm{S}_{n+1}^{\mathrm{P}}, \mathrm{E}_{n+1}^{\mathrm{P}}, \mathrm{R}_{n+1}^{\mathrm{P}}), \\ \mathrm{E}_{n+1} \approx \mathrm{E}(0) + &\frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \sum_{\ell=0}^{n} \Delta_{\ell,n+1} \mathcal{G}_2(t_\ell, \mathrm{S}_\ell, \mathrm{E}_\ell, \mathrm{R}_\ell) \\ &+ \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \mathcal{G}_2(t_{n+1}, \mathrm{S}_{n+1}^{\mathrm{P}}, \mathrm{E}_{n+1}^{\mathrm{P}}, \mathrm{R}_{n+1}^{\mathrm{P}}), \\ \mathrm{R}_{n+1} \approx \mathrm{R}(0) + &\frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \sum_{\ell=0}^{n} \Delta_{\ell,n+1} \mathcal{G}_3(t_\ell, \mathrm{S}_\ell, \mathrm{E}_\ell, \mathrm{R}_\ell) \\ &+ \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+2)} \mathcal{G}_3(t_{n+1}, \mathrm{S}_{n+1}^{\mathrm{P}}, \mathrm{E}_{n+1}^{\mathrm{P}}, \mathrm{R}_{n+1}^{\mathrm{P}}), \end{aligned} \tag{4.13}$$

where $\hbar = \frac{\mathrm{T}^{\sigma}}{\mathrm{N}}$, and $\mathrm{S}^{\mathrm{P}}_{n+1}, \mathrm{E}^{\mathrm{P}}_{n+1}$ and $\mathrm{R}^{\mathrm{P}}_{n+1}$ are defined as:

$$\begin{aligned}
\mathrm{S}^{\mathrm{P}}_{n+1} &\approx \mathrm{S}(0) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+1)} \sum_{\ell=0}^{n} \Theta_{\ell,n+1} \mathcal{G}_1(t_\ell, \mathrm{S}_\ell, \mathrm{E}_\ell, \mathrm{R}_\ell),\\
\mathrm{E}^{\mathrm{P}}_{n+1} &\approx \mathrm{E}(0) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+1)} \sum_{\ell=0}^{n} \Theta_{\ell,n+1} \mathcal{G}_2(t_\ell, \mathrm{S}_\ell, \mathrm{E}_\ell, \mathrm{R}_\ell),\\
\mathrm{R}^{\mathrm{P}}_{n+1} &\approx \mathrm{R}(0) + \frac{\sigma^{-\beta}\hbar^{\beta}}{\Gamma(\beta+1)} \sum_{\ell=0}^{n} \Theta_{\ell,n+1} \mathcal{G}_3(t_\ell, \mathrm{S}_\ell, \mathrm{E}_\ell, \mathrm{R}_\ell),
\end{aligned} \tag{4.14}$$

where $\Theta_{\ell,n+1} = [(n+1-\ell)^{\beta} - (n-\ell)^{\beta}]$, and $\mathcal{G}_1, \mathcal{G}_2$ and $\mathcal{G}_3$ are given as:

$$\begin{aligned}
&\mathcal{G}_1(t, \mathrm{S}, \mathrm{E}, \mathrm{R}) := \nu - (\nu + \alpha \mathrm{R})\mathrm{S},\\
&\mathcal{G}_2(t, \mathrm{S}, \mathrm{E}, \mathrm{R}) := \alpha \mathrm{SR} - \gamma \mathrm{E},\\
&\mathcal{G}_3(t, \mathrm{S}, \mathrm{E}, \mathrm{R}) := \lambda \mathrm{E} - (\lambda \mathrm{E} + \delta)\mathrm{R}.
\end{aligned} \tag{4.15}$$

5. Numerical results and discussion

In previous subsection, as we obtained the solution form of model (1.2). Now, we have presented the obtained numerical results *via* plots. In computation, all the parameters data are taken from [21], which are given as:

$$\nu = 0.0045,\ \alpha = 0.006,\ \gamma = 0.333,\ \lambda = 0.375,\ \delta = 0.02941.$$

Total population $\mathrm{N}_p = 5071126$, using this the initial values are defined as

$$\mathrm{S}(0) = \mathrm{N}_1 = \frac{5070822}{5071126} = 0.9999400528,$$

$$\mathrm{E}(0) = \mathrm{N}_2 = \frac{304}{5071126} = 0.0000599472,\ \mathrm{R}(0) = \mathrm{N}_3 = 0.1.$$

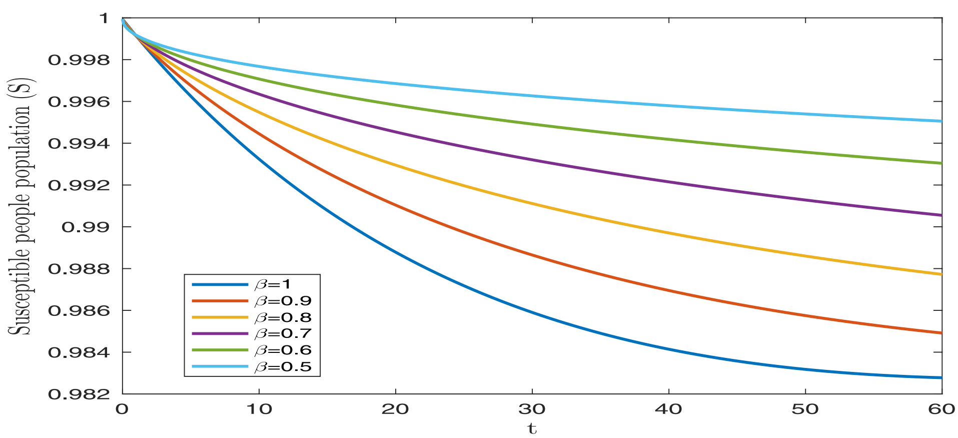

Figure 1. The dynamics of susceptible population at arbitrary β and $\sigma = 1.1$ for DFD model (1.2).

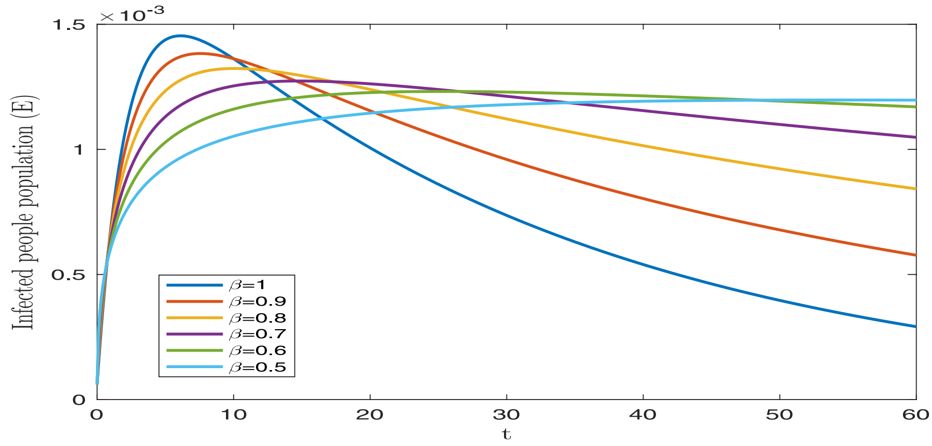

FIGURE 2. The dynamics of infected population at arbitrary β and $\sigma = 1.1$ for DFD model (1.2).

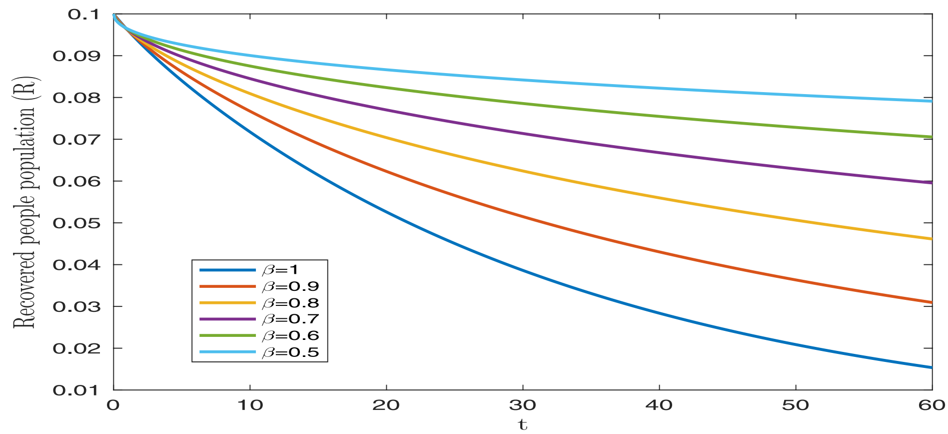

FIGURE 3. The dynamics of recovered population at arbitrary β and $\sigma = 1.1$ for DFD model (1.2).

All the plots Figures 1–6 are display the transmission dynamics of dengue fever in various subclasses, when $T = 60$ days and $N = 600$ for various values of the orders β and σ.

6. Conclusion

In this article, we analyzed an arbitrary order DFD model using generalized Caputo derivative. The existence of model and its uniqueness have been investigated with fixed point theory. Further, a new adaptive P-C algorithm was implemented for the solution of the DFD model. The simulations were presented at various fractional-order through graphics. The presented graphics show that the results depend on the fractional-order parameters β and σ. Finally, we can say the considered generalized fractional operator provides a better explanation of the system dynamics and can be applied for the analysis of other infectious diseases model.

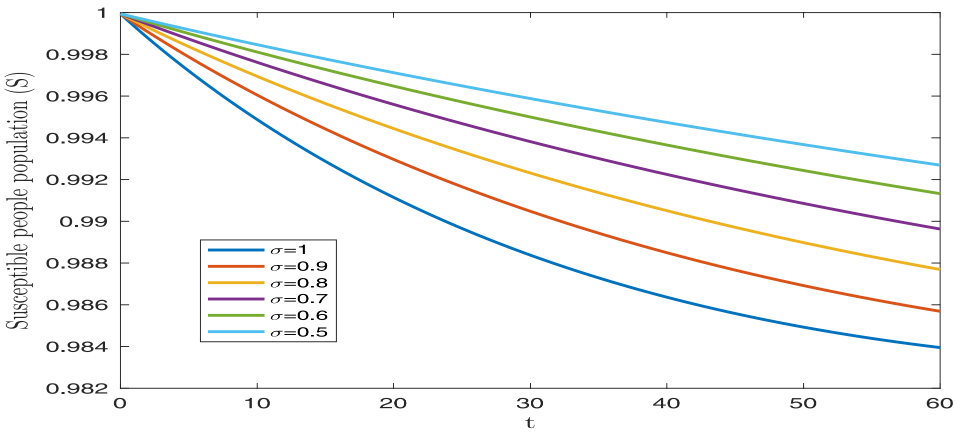

FIGURE 4. The dynamics of susceptible population at arbitrary σ and $\beta = 0.99$ for DFD model (1.2).

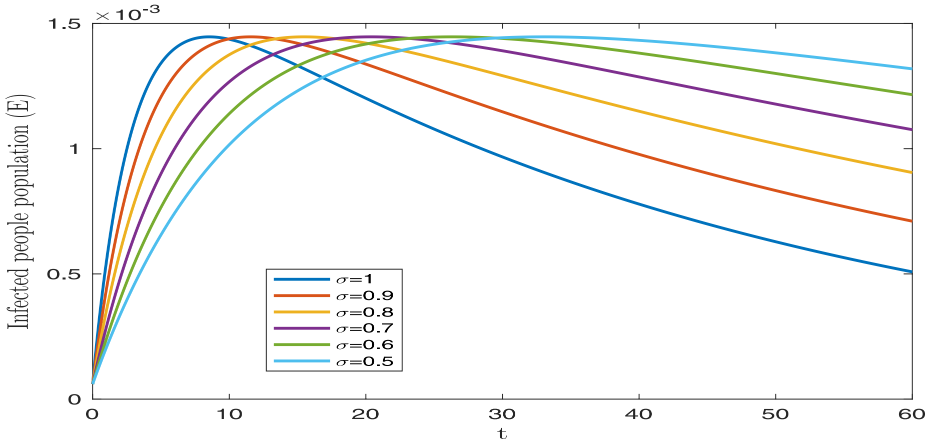

FIGURE 5. The dynamics of infected population at arbitrary σ and $\beta = 0.99$ for DFD model (1.2).

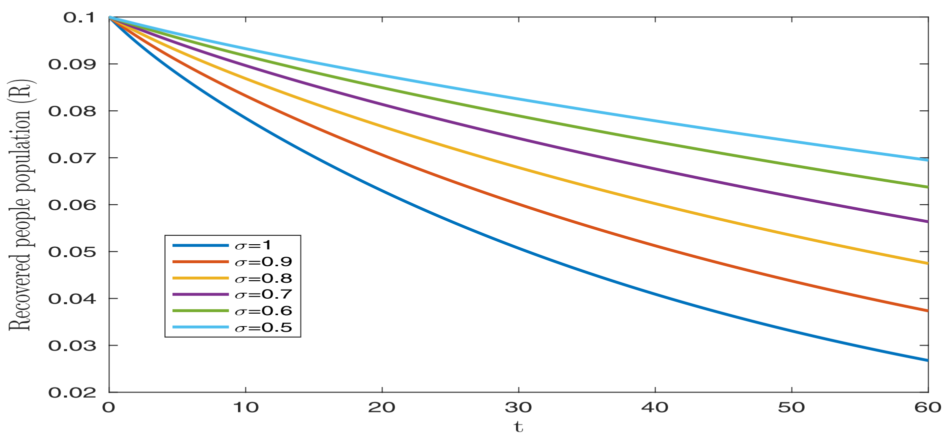

FIGURE 6. The dynamics of recovered population at arbitrary σ and $\beta = 0.99$ for DFD model (1.2).

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