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CONTROLLABILITY OF DELAYED DISCRET FORNASINI-MARCHESINI MODEL *VIA* QUANTIZATION AND RANDOM PACKET DROPOUTS

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Abstract. This research is devoted to Fornasnisi-Marchesini model (FM). More precisely, the investigation of the control problem for the second model discrete-time FM. The model takes into account the random packet loss and quantization errors in the network environment. So our modelling method has the potential to achieve a better stabilization effects. Random packet dropouts, time delays and quantization are taken into consideration in the feedback control problem simultaneously. Measured signals are quantized before being communicated. A logarithmic quantizer is utilized and quantized signal measurements are handled by a sector bound method. The random packet dropouts are modeled as a Bernoulli process. A control law model which depends on packet dropouts and quantization is formulated. Notably, we lighten the assumptions by using the Schur complement. Besides, both a state feedback controller and an observer-based output feedback controller are designed to ensure corresponding closed-loop systems asymptotically stability. Sufficient conditions on mean square asymptotic stability in terms of LMIs have been obtained. Finally, two numerical example show the feasibility of our theoretical results.

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1. Introduction

The most frequently used state-space models of two-dimensional (2-D) systems are: the Roesser (R) model, the first and the second order Fornasini-Marchesini models [8, 9, 11, 13, 19, 21, 22, 26–28, 34, 39].

On the other hand, the control and effect of delays of 2-D systems has been studied in [12, 13, 15, 19, 21, 23, 25, 27, 28, 41].

In [17], Galkowski and Rogers reviews the significant developments in the succeeding four decades in both theory and applications. The problem of Stability analysis of 2-D discrete systems has been investgated for the Fornasini-Marchesini second model with state saturation in [30]. Wan *et al.* has been proposed an iterative learning Control for the two-dimensional Linear Discrete Systems with Fornasini-Marchesini Model [33, 34].

Keywords and phrases: Quantization, Lyapunov-Krasovskii, Random Packet Dropouts, Fornasini-Marchesini (FMII) models.

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Xu *et al.* proposed in [38] a constructive realization procedure for 2D systems which may lead to a Fornasini-Marchesini local state-space model with much lower order than the existing realization procedure given by Bisiacco *et al.* In [10, 23, 24, 36, 40], the authors considers the effect of multiple quantizers and channel packet dropouts in state estimation of networked system. The control systems with packet loss has been studied in [29, 32, 35].

As a natural continuation of these works, a networked-based feedback control problem is investigated for 2-D systems with time-varying delays described by the FM second model involving quantization and packet dropouts in this research. The measured signals are quantized before being communicated.

Main contributions of this paper are summarized as below:

1a) A new feedback Control problem involving quantization, packet dropout and time delays is introduced for 2-D discrete-time systems.
1b) Quantization and packet dropout are introduced for 2-D discrete-time systems with time-varying delays, which are applied in controller design.
1c) Quantization and random packet dropouts are considered simultaneously in the feedback control problem for 2-D NCSs, where data packets may be missing with different probabilities in different directions.
2a) A control law model considering random packet dropout and quantization is proposed for 2-D discrete-time systems with time-varying delays.
2b) Stability conditions for the resulting 2-D closed-loop systems are established, which depend on time delays, quantization density and packet dropout probability.
3) An observer-based output feedback controller relying on packet dropouts and quantization for 2-D NCSs is designed. A new problem of observer-based output feedback control for 2-D discrete-time systems is studied.

We summarize the content of this paper as follows. Section 2 formulates the problem of 2-D discrete-time systems with time-varying delays described by the FM second model, and the 2-D discrete-time systems subject to random packet dropouts and quantization. A state feedback control problem is studied in Section 3. In Section 4, a problem of output feedback control based on observer is shown. Finally, two numerical examples are given in Section 5 to illustrate effectiveness of the proposed methods.

Notation: Throughout the paper, R^n represents the n-dimensional Euclidean space. For a real matrix P, P^T and P^{-1} denote the transpose and the inverse of P, respectively. A matrix $P > 0$, means that it is a symmetric, positive definite real matrix. The shorthand prob$(\cdot)$ means the occurrence probability of the event "$\cdot$". The shorthand diag$\{\cdot\}$ denotes a block diagonal matrix. The symmetric terms in a symmetric matrix are denoted by $*$. The notation $\|\cdot\|$ refers to the Euclidean vector norm. $E\{\cdot\}$ denotes the expectation.

2. Problem formulation

Consider a class of 2-D NCSs described as follows:

$$\begin{aligned} x(i+1,j+1) = A_1x(i+1,j) + A_2x(i,j+1) + A_{1d}x(i+1,j-d_1(j)) \\ +A_{2d}x(i-d_2(i),j+1) + B_1u(i+1,j) + B_2u(i,j+1), \end{aligned} \tag{2.1}$$

$$y(i,j) = Cx(i,j), \tag{2.2}$$

where $x(i,j) \in R^n$ is the state vector, $u(i,j) \in R^m$ is the control input vector, $y(i,j) \in R^q$ is the measured output vector, A_1, A_2, A_{1d}, A_{2d}, B_1 and B_2 are constant matrices with appropriate dimensions. $d_1(j)$ and $d_2(i)$ are time-varying delays along vertical direction and horizontal direction, respectively, satisfying:

$$0 < d_{1m} \le d_1(j) \le d_{1M}, 0 < d_{2m} \le d_2(i) \le d_{2M}. \tag{2.3}$$

The boundary conditions are assumed as:

$$\begin{aligned}&\{x(i,j)=\varphi_{i,j}\}, \forall i\geq 0, j=-d_{1M}, -d_{1M}+1,\dots,0,\\&\{x(i,j)=\psi_{i,j}\}, \forall j\geq 0, i=-d_{2M}, -d_{2M}+1,\dots,0,\\&\varphi_{0.0}=\psi_{0,0},\end{aligned} \tag{2.4}$$

with the functions $\varphi_{i,j}$ and $\psi_{i,j}$ satisfying:

$$\sum_{i=0}^{\infty}\sum_{j=-d_{1M}}^{0}\varphi_{i,j}^{T}\varphi_{i,j}<\infty, \sum_{j=0}^{\infty}\sum_{i=-d_{2M}}^{0}\psi_{i,j}^{T}\psi_{i,j}<\infty. \tag{2.5}$$

In this paper, the measurement signals will be quantized before transmission. A logarithmic quantizer as in [14] is applied in this paper. The set of quantized levels is described as $U=\{\pm u_i : u_i=\rho^i u_0, i=0,\pm1,\pm2,\cdots\}\bigcup\{0\}, 0<\rho<1, u_0>0$, where the parameter ρ denotes the quantization density [16]. The logarithmic quantizer $\rho(\cdot)$ is defined as follows:

$$q(v)=\begin{cases}u_i, & if\ \frac{1}{1+\delta}u_i<v\leq\frac{1}{1-\delta}u_i,\\ 0, & if\ v=0,\\ -q(-v), & if\ v<0,\end{cases} \tag{2.6}$$

with $\delta=\frac{1-\rho}{1+\rho}$.

Due to the network bandwidth is limited, it's inevitable that the intermittent data packet dropouts occur during transmission. Hence, a Bernoulli process $\alpha_{i,j}$ with the probability distribution is introduced as follows:

$$\text{prob}\{\alpha_{i,j}=1\}=E\{\alpha_{i,j}\}=\alpha, \tag{2.7}$$

$$\text{prob}\{\alpha_{i,j}=0\}=1-E\{\alpha_{i,j}\}=1-\alpha, \tag{2.8}$$

where $0\leq\alpha\leq1$ is a known constant. α denotes the possibility of packet loss. To introduce main results of this paper, the following lemma is given.

Lemma 2.1. *([37]) Given matrices Q, H and E of appropriate dimensions with Q being symmetrical, for all F satisfying $F^TF\leq I$, $Q+HFE+E^TF^TH^T<0$, if and only if there exists a scalar $\varepsilon>0$, such that*

$$Q+\varepsilon HH^T+\varepsilon^{-1}E^TE<0. \tag{2.9}$$

3. State feedback control

Assume that the state of system (2.1)–(2.2) is measurable in this section. The measured state has to be quantized before being transmitted to the controller. In addition, the random data packet dropouts exist in the network channel. Hence, the measurement of state signals is described as follows:

$$x_s(i,j)=\alpha_{i,j}q(x(i,j)), \tag{3.1}$$

where $x_s(i,j)$ is the signal received by the controller, $q(\cdot)$ is the logarithmic quantizer defined in condition (2.6). To investigate the problem of stability, the sector bound method [16] is adopted to deal with the quantization error:

$$q(x(i,j))-x(i,j)=\Delta(x(i,j))x(i,j), \|\Delta(x(i,j))\|\leq\delta. \tag{3.2}$$

The state feedback control law based on quantized state information and packet dropouts is described as:

$$u(i,j) = Kx_s(i,j), \tag{3.3}$$

where K is the controller gain to be designed.

Remark 3.1. With equality (3.1), the measurement $x_s(i,j)$ of state signals is no longer equivalent to the state vector $x(i,j)$ due to the effects of packet loss and quantization, which is inevitable in many applications. The system measurement model (3.1) can be used to represent missing measurements, which is first applied in 2-D systems with time delays in a networked environment.

Remark 3.2. The control law model (3.3) can be rewritten as:

$$u(i,j) = \alpha_{i,j}K(I + (\Delta x(i,j)))x(i,j). \tag{3.4}$$

The random packet dropouts and quantization errors in the network environment are taken into account by model (3.4). In the existing results [15, 21], the controller was designed without considering any network factors. The state feedback control law proposed in [9] only considers packet dropouts. Compared with the controllers in literature's [9, 15, 21], model (3.4) is more general and meaningful in the practical application.

Combining model (3.4) and system (2.1), the closed-loop system is available as follows:

$$\begin{aligned} x(i+1,j+1) = {} & A_1x(i+1,j) + A_2x(i,j+1) + A_{1d}x(i+1,j-d_1(j)) \\ & +A_{2d}x(i-d_2(i),j+1) \\ & +\alpha_{i+1,j}B_1K(1+\Delta(x(i+1,j)))x(i+1,j) \\ & +\alpha_{i,j+1}B_2K(1+\Delta(x(i,j+1)))x(i,j+1). \end{aligned} \tag{3.5}$$

Remark 3.3. Due to the introductions of the stochastic variable $\alpha_{i,j}$ and the quantization error $\Delta(x(i,j))$, the closed-loop system (3.5) is an uncertain stochastic system. Hence, problems considered in this paper have a broader significance and practical value than ones in [15, 27, 28].

Definition 3.4. ([31]) The system (2.1)–(2.2) with discrete time is said to be globally asymptotically stable in the mean square if every zero solution satisfies:

$$\lim_{\delta \longrightarrow +\infty} E\{\|x(\delta)\|^2 + \|y(\delta)\|^2\} = 0$$

Lemma 3.5. *(Shur complement [31]) Given constant matrices S_{11}, S_{12}, S_{21}, S_{22}. If we have $S_{11} = S_{11}^T$, $S_{12} = S_{21}^T$ and $S_{22} = S_{22}^T$, then*

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} < 0.$$

A stability condition for the closed-loop system (3.5) is given in the following theorem.

Theorem 3.6. *For given scalars d_{1m}, d_{2m}, d_{1M}, d_{2M}, quantization density $\rho > 0$ and packet-loss rate α, the 2-D closed-loop system (3.5) is asymptotically stable in the mean square if there exist matrices $P > 0$, $Q > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$, $R_1 > 0$, $R_2 > 0$, K and scalar $\varepsilon > 0$ satisfying:*

$$\begin{bmatrix} \Phi_{11} & * & * \\ \Phi_{21} & \Phi_{22} & * \\ 0 & \Phi_{32} & \Phi_{33} \end{bmatrix} < 0, \tag{3.6}$$

where

$$
\begin{aligned}
&\Phi_{11} = diag\{\phi_{11} + \delta^2\varepsilon, \phi_{22} + \delta^2\varepsilon, -R_1, -R_2, -Q_1, -Q_2, -Q_3, -Q_4\},\\
&\Phi_{21} = \begin{bmatrix} A_1 + \alpha B_1 K & A_2 + \alpha B_2 K & A_{1d} & A_{2d} & 0 & 0 & 0 & 0 \\ B_1 K & B_2 K & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\\
&\Phi_{22} = \begin{bmatrix} -P^{-1} & 0 \\ 0 & -(\alpha(1-\alpha))^{-1}P^{-1} \end{bmatrix},\\
&\Phi_{32} = \begin{bmatrix} \alpha K^T B_1^T & \alpha K^T B_2^T \\ K^T B_1^T & K^T B_2^T \end{bmatrix},\\
&\Phi_{33} = \begin{bmatrix} -\varepsilon I & 0 \\ 0 & -\varepsilon I \end{bmatrix},\\
&\phi_{11} = -Q + Q_1 + Q_3 + (d_{1M} - d_{1m} + 1)R_1,\\
&\phi_{22} = Q - P + Q_2 + Q_4 + (d_{2M} - d_{2m} + 1)R_2.
\end{aligned}
$$

Proof. Denote

$$x_{\xi,\eta} = x(i + \xi, j + \eta).$$

Choose the following Lyapunov-Krasovskii functional as:

$$
\begin{aligned}
V_{11} &= V_{11}^{(1)} + V_{11}^{(2)} + V_{11}^{(3)} + V_{11}^{(4)},\\
V_{11}^{(1)} &= x_{1,1}^T P x_{1,1},\\
V_{11}^{(2)} &= \sum_{l=-d_{1M}}^{-1} x_{1,l+1}^T Q_1 x_{1,l+1} + \sum_{l=-d_{2M}}^{-1} x_{l+1,1}^T Q_2 x_{l+1,1},\\
V_{11}^{(3)} &= \sum_{l=-d_{1m}}^{-1} x_{1,l+1}^T Q_3 x_{1,l+1} + \sum_{l=-d_{2m}}^{-1} x_{l+1,1}^T Q_4 x_{l+1,1},\\
V_{11}^{(4)} &= \sum_{\theta=-d_{1M}}^{-d_{1m}} \sum_{l=\theta}^{-1} x_{1,l+1}^T R_1 x_{1,l+1} + \sum_{\theta=-d_{2M}}^{-d_{2m}} \sum_{l=\theta}^{-1} x_{l+1,1}^T R_2 x_{l+1,1}.
\end{aligned}
$$

Let $v_{1,0} = \{x_{1,0}, x_{1,-1}, \cdots, x_{1,-d_{1M}}\}, v_{0,1} = \{x_{0,1}, x_{-1,1}, \ldots, x_{-d_{2M},1}\}$, along the trajectory of system (3.5), calculating the difference of V_{11} and taking the mathematical expectation $E\{\Delta V^{(k)}\} = E\{V_{11}^{(k)} | v_{1,0}, v_{0,1}\} - V_{d_1}^{(k)} - V_{d_2}^{(k)}$, $k = 1, 2, 3, 4$, with:

$$
\begin{aligned}
V_{d_1} &= V_{d_1}^{(1)} + V_{d_1}^{(2)} + V_{d_1}^{(3)} + V_{d_1}^{(4)},\\
V_{d_1}^{(1)} &= x_{1,0}^T Q x_{1,0},\\
V_{d_1}^{(2)} &= \sum_{l=-d_{1M}}^{-1} x_{1,l}^T Q_1 x_{1,l},\\
V_{d_1}^{(3)} &= \sum_{l=-d_{1m}}^{-1} x_{1,l}^T Q_3 x_{1,l},
\end{aligned}
$$

$$
\begin{aligned}
V_{d_1}^{(4)} &= \sum_{\theta=-d_{1M}}^{-d_{1m}} \sum_{l=\theta}^{-1} x_{1,l}^T R_1 x_{1,l},\\
V_{d_2} &= V_{d_2}^{(1)} + V_{d_2}^{(2)} + V_{d_2}^{(3)} + V_{d_2}^{(4)},\\
V_{d_2}^{(1)} &= x_{0,1}^T (P-Q) x_{0,1},\\
V_{d_2}^{(2)} &= \sum_{l=-d_{2M}}^{-1} x_{l,1}^T Q_2 x_{l,1},\\
V_{d_2}^{(3)} &= \sum_{l=-d_{2m}}^{-1} x_{l,1}^T Q_4 x_{l,1},\\
V_{d_2}^{(4)} &= \sum_{\theta=-d_{2M}}^{-d_{2m}} \sum_{l=\theta}^{-1} x_{l,1}^T R_2 x_{l,1}.
\end{aligned}
$$

It is shown as:

$$
\begin{aligned}
E\big\{\Delta V^{(1)}\big\} &= [A_1 x_{1,0} + A_2 x_{0,1} + A_{1d} x_{1,-d_1(j)} + A_{2d} x_{-d_2(i),1}\\
&\quad +\alpha B_1 K(I+\Delta(x_{1,0})x_{1,0} + \alpha B_2 K(I+\Delta(x_{0,1})x_{0,1}]^T P[A_1 x_{1,0}\\
&\quad +A_2 x_{0,1} + A_{1d} x_{1,-d_1(j)} + A_{2d} x_{-d_2(i),1} + \alpha B_1 K(I+\Delta(x_{1,0})x_{1,0}\\
&\quad +\alpha B_2 K(I+\Delta(x_{0,1})x_{0,1}] + (1-\alpha)\alpha[B_1 K(I+\Delta(x_{1,0})x_{1,0}\\
&\quad +B_2 K(I+\Delta(x_{0,1})x_{0,1}]^T P[B_1 K(I+\Delta(x_{1,0})x_{1,0}\\
&\quad +B_2 K(I+\Delta(x_{0,1})x_{0,1}]\\
&\quad -x_{1,0}^T Q x_{1,0} - x_{0,1}^T (P-Q) x_{0,1}, \qquad (3.7)\\
E\big\{\Delta V^{(2)}\big\} &= x_{1,0}^T Q_1 x_{1,0} + x_{0,1}^T Q_2 x_{0,1} - x_{1,-d_{1M}}^T Q_1 x_{1,-d_{1M}}\\
&\quad -x_{-d_{2M},1}^T Q_2 x_{-d_{2M},1}, \qquad (3.8)\\
E\big\{\Delta V^{(3)}\big\} &= x_{1,0}^T Q_3 x_{1,0} + x_{0,1}^T Q_4 x_{0,1} - x_{1,-d_{1m}}^T Q_3 x_{1,-d_{1m}}\\
&\quad -x_{-d_{2m},1}^T Q_4 x_{-d_{2m},1}, \qquad (3.9)\\
E\big\{\Delta V^{(4)}\big\} &\le (d_{1M} - d_{1m} + 1) x_{1,0}^T R_1 x_{1,0} + (d_{2M} - d_{2m} + 1) x_{0,1}^T R_2 x_{0,1}\\
&\quad -x_{1,-d_{1(j)}}^T R_1 x_{1,-d_{1(j)}} - x_{-d_{2(i)},1}^T R_2 x_{-d_{2(i)},1}. \qquad (3.10)
\end{aligned}
$$

By conditions (3.7)–(3.10), it can be shown that:

$$
E\{\Delta V\} = \sum_{k=1}^{4} E\Big\{\Delta V^{(k)}\Big\} \le \xi^T \Omega \xi, \qquad (3.11)
$$

where

$$
\begin{aligned}
\xi &= \Big[\, x_{1,0}^T \; x_{0,1}^T \; x_{1,-d_1(j)}^T \; x_{-d_2(i),1}^T \; x_{1,-d_{1M}}^T \; x_{-d_{2M},1}^T \; x_{1,-d_{1m}}^T \; x_{1,-d_{2m}}^T \,\Big]^T,\\
\Omega &= V^T M_1 V - M_2,\\
V &= \begin{bmatrix} A_1 + \alpha B_1 K(I+\Delta(x_{1,0}) & A_2 + \alpha B_2 K(I+\Delta(x_{0,1}) & A_{1d} & A_{2d} & 0 & 0 & 0 & 0\\ B_1 K(I+\Delta(x_{1,0}) & B_2 K(I+\Delta(x_{0,1}) & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\end{aligned}
$$

$$M_1 = \begin{bmatrix} P & 0 \\ 0 & (\alpha(1-\alpha))P \end{bmatrix},$$
$$M_2 = diag\{\phi_{11}, \phi_{22}, -R_1, -R_2, -Q_1, -Q_2, -Q_3, -Q_4\}.$$

According to Schur's complement, $\Omega < 0$ is equivalent to:

$$\begin{bmatrix} M_2 & * \\ V & -M_1^{-1} \end{bmatrix} < 0. \tag{3.12}$$

The matrix inequality (3.12) can be rewritten as:

$$\begin{bmatrix} \Phi_{11} & * \\ \Phi_{21} & \Phi_{22} \end{bmatrix} + HG + G^T H^T < 0, \tag{3.13}$$

where

$$H = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,0 & \alpha K^T B_1^T & \alpha K^T B_2^T \\ 0\,0\,0\,0\,0\,0\,0\,0 & K^T B_1^T & K^T B_2^T \end{bmatrix}^T,$$
$$G = \begin{bmatrix} \Delta(x_{1,0}) & 0 & 0\,0\,0\,0\,0\,0\,0\,0 \\ 0 & \Delta(x_{0,1}) & 0\,0\,0\,0\,0\,0\,0\,0 \end{bmatrix}.$$

According to lemma 2.1, the matrix inequality (3.6) is obtained for any scalar $\varepsilon > 0$. Therefore, if the matrix inequality (3.6) holds, then $E\{\Delta V\} < 0$. From Lyapunov stability theory, the closed system (3.5) is asymptotically stable in the mean square. □

Remark 3.7. It should be noted that the matrix inequality (3.6) is not a strict linear matrix inequality (LMI). In general, there are two ways to deal with this problem. One is the cone complementarity linearization (CCL) algorithm [18], and another one is the method of variable substitution. CCL is an iterative algorithm involving complicated computation, and the solution obtained is suboptimal. Hence, in this paper, the variable substitution method is choosed to deal with the nonlinear terms. Thus Theorem 3.8 is given.

Theorem 3.8. *Under the assumptions as Theorem 3.6, the closed-loop system (3.5) is asymptotically stable in the mean square, if there exist matrices $\widetilde{P} > 0, \widetilde{Q} > 0, \widetilde{Q}_1 > 0, \widetilde{Q}_2 > 0, \widetilde{Q}_3 > 0, \widetilde{Q}_4 > 0, \widetilde{R}_1 > 0, \widetilde{R}_2 > 0, W > 0$ and $\widetilde{K}$ satisfying the following LMI as:*

$$\begin{bmatrix} \widetilde{\Phi}_{11} & * & * \\ \widetilde{\Phi}_{21} & \widetilde{\Phi}_{22} & * \\ 0 & \widetilde{\Phi}_{32} & \widetilde{\Phi}_{33} \end{bmatrix} < 0, \tag{3.14}$$

where

$$\widetilde{\Phi}_{11} = diag\Big\{\widetilde{\phi}_{11} + \delta^2 W, \widetilde{\phi}_{22} + \delta^2 W, -\widetilde{R}_1, -\widetilde{R}_2, -\widetilde{Q}_1, -\widetilde{Q}_2, -\widetilde{Q}_3, -\widetilde{Q}_4\Big\},$$
$$\widetilde{\Phi}_{22} = \begin{bmatrix} -\widetilde{P} & 0 \\ 0 & -(\alpha(1-\alpha))^{-1}\widetilde{P} \end{bmatrix},$$
$$\widetilde{\Phi}_{21} = \begin{bmatrix} A_1\widetilde{P} + \alpha B_1\widetilde{K} & A_2\widetilde{P} + \alpha B_2\widetilde{K} & A_{1d}\widetilde{P} & A_{2d}\widetilde{P} & 0\,0\,0\,0 \\ B_1\widetilde{K} & B_1\widetilde{K} & 0 & 0 & 0\,0\,0\,0 \end{bmatrix},$$

$$\widetilde{\Phi}_{32} = \begin{bmatrix} \alpha\widetilde{K}^T B_1^T & \alpha\widetilde{K}^T B_2^T \\ \widetilde{K}^T B_1^T & \widetilde{K}^T B_2^T \end{bmatrix},$$
$$\widetilde{\Phi}_{33} = \begin{bmatrix} -W & 0 \\ 0 & -W \end{bmatrix},$$
$$\widetilde{\phi}_{11} = -\widetilde{Q} + \widetilde{Q}_1 + \widetilde{Q}_3 + (d_{1M} - d_{1m} + 1)\widetilde{R}_1,$$
$$\widetilde{\phi}_{22} = \widetilde{Q} - \widetilde{P} + \widetilde{Q}_2 + \widetilde{Q}_4 + (d_{2M} - d_{2m} + 1)\widetilde{R}_2.$$

The control gain in model (3.3) *is given by:*

$$K = \widetilde{K}\widetilde{P}^{-1}. \tag{3.15}$$

Proof. Let $\widetilde{P} = P^{-1}$, $\widetilde{Q} = P^{-1}QP^{-1}$, $\widetilde{Q}_1 = P^{-1}Q_1P^{-1}$, $\widetilde{Q}_2 = P^{-1}Q_2P^{-1}$, $\widetilde{Q}_3 = P^{-1}Q_3P^{-1}$, $\widetilde{Q}_4 = P^{-1}Q_4P^{-1}$, $\widetilde{R}_1 = P^{-1}R_1P^{-1}$, $\widetilde{R}_2 = P^{-1}R_2P^{-1}$, $\widetilde{K} = KP^{-1}$, $W = P^{-1}(\varepsilon I)P^{-1}$, pre- and post-multiplying the matrix inequality (3.6) by $diag\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, P^{-1}, P^{-1}\}$, the matrix inequality (3.14) is obtained. Hence, the control gain is solved as $K = \widetilde{K}\widetilde{P}^{-1}$ from $\widetilde{K} = KP^{-1}$. The proof is completed. □

4. Observer-based output feedback control

In this section, we assume that only the output information of the system (2.1)–(2.2) is available and will be quantized before it is transmitted to the controller *via* communication network. Considering the data packet dropouts, it is given as:

$$y_s(i,j) = \alpha_{i,j} q(y(i,j), \tag{4.1}$$

where $y_s(i,j)$ is the estimation of output measurements at the side of controller. $q(\cdot)$ is the logarithmic quantizer defined as in condition (2.6), the quantized output measurements can be described as:

$$q(y(i,j)) - y(i,j) = \Delta(y(i,j))y(i,j), \|\Delta(y(i,j)\| \leq \delta. \tag{4.2}$$

The following observer-based output feedback controller is constructed as:

$$\begin{aligned} \widehat{x}(i+1,j+1) = {} & A_1\widehat{x}(i+1,j) + A_2\widehat{x}(i,j+1) + B_1u(i+1,j) \\ & +B_2u(i,j+1) + L[y_s(i+1,j) - C\widehat{x}(i+1,j)] \\ & +L[y_s(i,j+1) - C\widehat{x}(i,j+1)], \end{aligned} \tag{4.3}$$
$$u(i,j) = K\widehat{x}(i,j), \tag{4.4}$$

where the gain matrices L and K are to be designed later.

An observer is utilized due to the states of the 2-D systems are usually not adequately measured in practice. In order to fully consider the effects of time delays, packet loss and quantization, the error system is established.

Remark 4.1. Due to the system states are usually not adequately measured in practice. An observer is utilized to get more accurate state estimates. Then, the controller achieves the control of 2-D systems (2.1)–(2.2) relying on these estimates. In the observer-based output feedback control problem, random packet dropouts and quantization are also considered.

Define the error as $e(i,j) = x(i,j) - \widehat{x}(i,j)$, the error system is obtained as follows:

$$e(i+1,j+1) = (A_1 - LC)e(i+1,j) + (A_2 - LC)e(i,j+1)$$

$$
\begin{aligned}
&+A_{1d}x(i+1,j-d_1(j))+A_{2d}x(i-d_2(i),j+1)\\
&+(1-\alpha_{i+1,j}(I+\Delta(y(i+1,j))))LCx(i+1,j)\\
&+(1-\alpha_{i,j+1}(I+\Delta(y(i,j+1))))LCx(i,j+1).
\end{aligned}
\tag{4.5}
$$

Moreover, defining $\eta(i,j)=\left[x(i,j)^T e(i,j)^T\right]^T$, the following augmented error dynamic system can be obtained as:

$$
\begin{aligned}
\eta(i+1,j+1)=\bar{A}_1\eta(i+1,j)+\bar{A}_2\eta(i,j+1)+\bar{A}_{1d}x(i+1,j-d_1(j))\\
+\bar{A}_{2d}x(i-d_2(i),j+1),
\end{aligned}
\tag{4.6}
$$

with

$$
\begin{aligned}
\bar{A}_1&=\begin{bmatrix} A_1+B_1K & -B_1K \\ (1-\alpha_{i+1,j}(I+\Delta(y(i+1,j))))LC & A_1-LC\end{bmatrix},\\
\bar{A}_2&=\begin{bmatrix} A_2+B_2K & -B_2K \\ (1-\alpha_{i,j+1}(I+\Delta(y(i,j+1))))LC & A_2-LC\end{bmatrix},\\
\bar{A}_{1d}^T&=\begin{bmatrix} A_{1d}^T \; A_{1d}^T\end{bmatrix},\\
\bar{A}_{2d}^T&=\begin{bmatrix} A_{2d}^T \; A_{2d}^T\end{bmatrix}.
\end{aligned}
$$

The problem of output feedback controller design based on observer will be studied in the following theorem.

Theorem 4.2. *For given scalars d_{1m}, d_{2m}, d_{1M}, d_{2M}, quantization density $\rho>0$ and packet-loss rate α, the 2-D closed systerm (4.6) is asymptotically stable in the mean square if there exist matrices $P>0$, $Q>0$, $M>0$, $N>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $Q_4>0$, $R_1>0$, $R_2>0$, K, L and scalar $\varepsilon>0$ satisfying:*

$$
\begin{bmatrix} \Psi_{11} & * & * \\ \Psi_{21} & \Psi_{22} & * \\ 0 & \Psi_{32} & \Phi_{33}\end{bmatrix}<0,
\tag{4.7}
$$

where

$$
\begin{aligned}
\Psi_{11}&=diag\{\phi_{11}+\delta^2\varepsilon,\phi_{22}+\delta^2\varepsilon,-N,N-M,-R_1,-R_2,-Q_1,-Q_2,-Q_3,\\
&\qquad -Q_4\},\\
\Psi_{22}&=diag\{-P^{-1},-M^{-1},-(\alpha(1-\alpha))^{-1}M^{-1}\},\\
\Psi_{21}&=\begin{bmatrix} A_1+B_1K & A_2+B_2K & -B_1K & -B_2K & A_{1d} & A_{2d} & 0\,0\,0\,0\\ (1-\alpha)LC & (1-\alpha)LC & A_1-LC & A_2-LC & A_{1d} & A_{2d} & 0\,0\,0\,0\\ LC & LC & 0 & 0 & 0 & 0 & 0\,0\,0\,0\end{bmatrix},\\
\Psi_{32}&=\begin{bmatrix} 0 & -\alpha LC & -\alpha LC\\ 0 & LC & LC\end{bmatrix}.
\end{aligned}
$$

Proof. Choose the Lyapunov function candidate as:

$$
\begin{aligned}
V_{11}&=V_{11}^{(1)}+V_{11}^{(2)}+V_{11}^{(3)}+V_{11}^{(4)}+V_{11}^{(5)},\\
V_{11}^{(1)}&=x_{1,1}^T P x_{1,1},\\
V_{11}^{(2)}&=e_{1,1}^T M e_{1,1},
\end{aligned}
$$

$$
\begin{aligned}
V_{11}^{(3)} &= \sum_{l=-d_{1M}}^{-1} x_{1,l+1}^T Q_1 x_{1,l+1} + \sum_{l=-d_{2M}}^{-1} x_{l+1,1}^T Q_2 x_{l+1,1},\\
V_{11}^{(4)} &= \sum_{l=-d_{1m}}^{-1} x_{1,l+1}^T Q_1 x_{1,l+1} + \sum_{l=-d_{2m}}^{-1} x_{l+1,1}^T Q_2 x_{l+1,1},\\
V_{11}^{(5)} &= \sum_{\theta=-d_{1M}}^{-d_{1m}} \sum_{l=\theta}^{-1} x_{1,l+1}^T R_1 x_{1,l+1} + \sum_{\theta=-d_{2M}}^{-d_{2m}} \sum_{l=\theta}^{-1} x_{l+1,1}^T R_2 x_{l+1,1}.
\end{aligned}
$$

Let $\eta_{1,0} = \{e_{1,0}, x_{1,0}, x_{1,-1}, \ldots, x_{1,-d_{1M}}\}$, $\eta_{0,1} = \{e_{0,1}, x_{0,1}, x_{-1,1}, \cdots, x_{-d_{2M},1}\}$, take the mathematical expectation $E\{\Delta V^{(k)}\} = E\{V_{11}^{(k)} | \eta_{1,0}, \eta_{0,1}\} - V_{d_1}^{(k)} - V_{d_2}^{(k)}$, $k = 1, 2, 3, 4, 5$, with:

$$
\begin{aligned}
&V_{d_1} = V_{d_1}^{(1)} + V_{d_1}^{(2)} + V_{d_1}^{(3)} + V_{d_1}^{(4)} + V_{d_1}^{(5)},\\
&V_{d_1}^{(1)} = x_{1,0}^T Q x_{1,0},\\
&V_{d_1}^{(2)} = e_{1,0}^T N e_{1,0},\\
&V_{d_1}^{(3)} = \sum_{l=-d_{1M}}^{-1} x_{1,l}^T Q_1 x_{1,l},\\
&V_{d_1}^{(4)} = \sum_{l=-d_{1m}}^{-1} x_{1,l}^T Q_1 x_{1,l},\\
&V_{d_1}^{(5)} = \sum_{\theta=-d_{1M}}^{-d_{1m}} \sum_{l=\theta}^{-1} x_{1,l}^T R_1 x_{1,l},\\
&V_{d_2} = V_{d_2}^{(1)} + V_{d_2}^{(2)} + V_{d_2}^{(3)} + V_{d_2}^{(4)} + V_{d_2}^{(5)},\\
&V_{d_2}^{(1)} = x_{0,1}^T (P - Q) x_{0,1},\\
&V_{d_2}^{(2)} = e_{0,1}^T (M - N) e_{0,1},\\
&V_{d_2}^{(3)} = \sum_{l=-d_{2M}}^{-1} x_{l,1}^T Q_2 x_{l,1},\\
&V_{d_2}^{(4)} = \sum_{l=-d_{2m}}^{-1} x_{l,1}^T Q_2 x_{l,1},\\
&V_{d_2}^{(5)} = \sum_{\theta=-d_{2M}}^{-d_{2m}} \sum_{l=\theta}^{-1} x_{l,1}^T R_2 x_{l,1}.
\end{aligned}
$$

According to a similar proof of Theorem 3.6, it is obtained that:

$$
E\{\Delta V\} = \sum_{k=1}^{5} E\{\Delta V^{(k)}\} \leq \varsigma^T \Lambda \varsigma, \tag{4.8}
$$

where

$$
\begin{aligned}
&\varsigma = \left[x_{1,0}^T \; x_{0,1}^T \; e_{1,0}^T \; e_{0,1}^T \; x_{1,-d_1(j)}^T \; x_{-d_2(i),1}^T \; x_{1,-d_{1M}}^T \; x_{-d_{2M},1}^T \; x_{1,-d_{1m}}^T \; x_{-d_{2m},1}^T \right]^T, \\
&\Lambda = F^T E_1 F - E_2, \\
&E_1 = diag\{P, M, (\alpha(1-\alpha))M\}, \\
&E_2 = diag\{\phi_{11}, \phi_{22}, -N, N-M, -R_1, -R_2, -Q_1, -Q_2, -Q_3, -Q_4\}, \\
&F = \begin{bmatrix} A_1 + B_1 K & A_2 + B_2 K & -B_1 K & -B_2 K \\ (1-\alpha(I-\Delta(x_{1,0}))LC & (1-\alpha(I-\Delta(x_{0,1})))LC & A_1 - LC & A_2 - LC \\ (I+\Delta(x_{1,0}))LC & (I+\Delta(x_{0,1}))LC & 0 & 0 \end{bmatrix. \\
&\qquad \left. \begin{matrix} A_{1d} & A_{2d} & 0 & 0 & 0 & 0 \\ A_{1d} & A_{2d} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right].
\end{aligned}
$$

By Schur's complement and Lemma 1, it is ensured that $\Lambda < 0$ by the matrix inequality (4.7), *i.e.*, $E\{\Delta V\} < 0$. Therefore, the closed-loop system (4.2) is asymptotically stable in the mean square. □

Due to condition (4.7) is not a LMI, the following Theorem is presented to linearize the condition (4.7). Without loss of generality, the matrix C is assumed to be of full column rank.

Theorem 4.3. *Under the assumptions as Theorem 4.2, the closed-loop system (4.6) is asymptotically stable in the mean square, if there exist matrices* $\widetilde{P} > 0, \widetilde{Q} > 0, \widetilde{M} > 0, \widetilde{N} > 0, \widetilde{Q}_1 > 0, \widetilde{Q}_2 > 0, \widetilde{Q}_3 > 0, \widetilde{Q}_4 > 0, \widetilde{R}_1 > 0, \widetilde{R}_2 > 0, W > 0, \widetilde{K}$ *and* $\widetilde{L}$ *satisfying the following conditions:*

$$
\begin{bmatrix} \widetilde{\Psi}_{11} & * & * \\ \widetilde{\Psi}_{21} & \widetilde{\Psi}_{22} & * \\ 0 & \widetilde{\Psi}_{32} & \widetilde{\Phi}_{33} \end{bmatrix} < 0, \tag{4.9}
$$

$$
C\widetilde{P} = \widetilde{P}_0 C, \tag{4.10}
$$

where

$$
\begin{aligned}
&\widetilde{\Psi}_{11} = diag\Big\{\widetilde{\phi}_{11} + \delta^2 W, \widetilde{\phi}_{22} + \delta^2 W, -\widetilde{N}, \widetilde{N} - \widetilde{M}, -\widetilde{R}_1, -\widetilde{R}_2, -\widetilde{Q}_1, -\widetilde{Q}_2, \\
&\qquad\qquad -\widetilde{Q}_3, -\widetilde{Q}_4\Big\}, \\
&\widetilde{\Psi}_{21} = \begin{bmatrix} A_1\widetilde{P} + B_1\widetilde{K} & A_2\widetilde{P} + B_2\widetilde{K} & -B_1\widetilde{K} & -B_2\widetilde{K} \\ (1-\alpha)\widetilde{L}C & (1-\alpha)\widetilde{L}C & A_1\widetilde{P} - \widetilde{L}C & A_2\widetilde{P} - \widetilde{L}C \\ \widetilde{L}C & \widetilde{L}C & 0 & 0 \end{bmatrix. \\
&\qquad \left. \begin{matrix} A_{1d}\widetilde{P} & A_{2d}\widetilde{P} & 0 & 0 & 0 & 0 \\ A_{1d}\widetilde{P} & A_{2d}\widetilde{P} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{matrix} \right], \\
&\widetilde{\Psi}_{22} = diag\Big\{-\widetilde{P}, -\widetilde{M}, -(\alpha(1-\alpha))^{-1}\widetilde{M}\Big\}, \\
&\widetilde{\Psi}_{32} = \begin{bmatrix} -\alpha C^T\widetilde{L}^T & -\alpha C^T\widetilde{L}^T \\ -C^T\widetilde{L}^T & -C^T\widetilde{L}^T \end{bmatrix}.
\end{aligned}
$$

The gain matrices in (4.3) *and* (4.4) *are given as:*

$$K = \widetilde{K}\widetilde{P}^{-1}, L = \widetilde{L}\widetilde{P}_0^{-1}. \tag{4.11}$$

Proof. Let $\widetilde{P} = P^{-1}, \widetilde{Q} = P^{-1}QP^{-1}, \widetilde{M} = P^{-1}MP^{-1}, \widetilde{N} = P^{-1}NP^{-1}, \widetilde{Q}_1 = P^{-1}Q_1P^{-1}, \widetilde{Q}_2 = P^{-1}Q_2P^{-1}, \widetilde{Q}_3 = P^{-1}Q_3P^{-1}, \widetilde{Q}_4 = P^{-1}Q_4P^{-1}, \widetilde{R}_1 = P^{-1}R_1P^{-1}, \widetilde{R}_2 = P^{-1}R_2P^{-1}, \widetilde{K} = KP^{-1}, W = P^{-1}(\varepsilon I)P^{-1}$, pre- and post-multiplying the matrix inequality (4.7) by $diag\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, P^{-1}, P^{-1}\}$ which is equivalent to the matrix inequality (4.9) by $\widetilde{L} = L\widetilde{P}_0$. Further, since the matrix C is full column rank, it can be shown as in [42] that:

$$rank(\widetilde{P}_0) \geq rank(\widetilde{P}_0C) = rank(C\widetilde{P}) \geq rank(C) = n, \tag{4.12}$$

which means that the matrix $\widetilde{P}_0$ is non-singular. The gain matrices in (4.3) and (4.4) are obtained as follows:

$$K = \widetilde{K}\widetilde{P}^{-1}, L = \widetilde{L}\widetilde{P}_0^{-1}. \tag{4.13}$$

The proof is completed. □

Remark 4.4. Theorem 4.3 provides an approach to solve the observer-based output feedback control problem for 2-D discrete-time systems (2.1)–(2.2), where the random packet dropouts and quantization are considered. The output feedback controller and observer (4.3)–(4.4) are constructed to ensure the closed-loop system (4.6) is asymptotically stable in the mean square. The main results are in the form of LMIs by the variable substitution method.

5. Numerical example

Some examples are given to illustrate the effectiveness of the main results developed in this paper.

Example 5.1. The model of the thermal processes in chemical reactors, heat exchanges and pipe furnaces can be expressed in a partial differential equation with time delays, is introduced in the 2-D FM model [39]. Consider the 2-D discrete system (2.1) with the following parameters:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.1 & 0.6 \\ 0 & 0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A_{1d} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\ A_{2d} &= \begin{bmatrix} 0 & 0 \\ 0 & -0.12 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}. \end{aligned} \tag{5.1}$$

The system states are assumed to be available, a state feedback controller will be designed. Let $\alpha = 0.8, \rho = 0.8, u_0 = 0.1$, the time-varying state delay satisfying $1 \leq d_2(i) \leq 20$. By solving the matrix inequality (3.14), it is shown as:

$$K = \begin{bmatrix} 0.0077 & -0.6489 \end{bmatrix}. \tag{5.2}$$

Under the consideration of random packet loss and quantization error, system (2.1) with matrices (5.1) is asymptotically stable, which means that the consideration in this paper is more rigorous and the proposed result is effective.

Example 5.2. In Example 1, only one direction of time-varying delays is considered. In order to prove that the stability results obtained in this paper have wider application in practice, the 2-D system (2.1)–(2.2) with the following parameters is studied:

$$\begin{aligned}
&A_1 = \begin{bmatrix} 0.1 & 0 \\ -1 & -0.2 \end{bmatrix}, A_2 = \begin{bmatrix} 0.4 & 0 \\ -0.2 & -0.1 \end{bmatrix}, A_{1d} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, \\
&A_{2d} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0.1 \\ 0.3 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0.1 & 0 \end{bmatrix}, \\
&C = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}.
\end{aligned} \tag{5.3}$$

The system states are assumed to be unavailable. An output feedback controller will be designed. The probability of the data packet missing is 20%, the parameters in the quantizer are the same as ones in example 1, and the time-varying state delays satisfying $1 \leq d_1(j) \leq 11, 1 \leq d_2(i) \leq 20$. Applying Theorem 4.3, it is given that:

$$K = \begin{bmatrix} 0.4402 & 0.1150 \\ -0.0872 & 0.0159 \end{bmatrix}, L = \begin{bmatrix} 0.1931 & -1.9318 \\ -0.4620 & 4.5052 \end{bmatrix}.$$

In the initial stage, the state curves have notable variations. This effect will gradually reduced when the system states asymptotically tend to zero. Thus, the stability of the given systems can be verified by the method proposed in this paper.

6. Conclusion

In this paper, the problem of feedback control for the 2-D discrete-time systems in a networked environment has been studied. Taking the time-varying delays, quantization and random packet dropouts into consideration simultaneously, the resulting closed-loop system has been constructed. The random packet dropouts have been modeled as a Bernoulli process. The quantization error has been treated as uncertainty which is sector-bounded. Sufficient conditions on asymptotic stability in terms of LMIs have been obtained. The state feedback controller, which is dependent on multiple network status, has been designed. Moreover, considering the states of actual systems are usually not fully measured, an observer-based output feedback control problem has been concerned to stabilize the original system. Finally, two examples have been presented to illustrate the availability of the proposed results. In the future, this results can be extended to other models: such as the class of neural networks models [1–7, 20]. Besides, we can think on the research of algebraic conditions and compare it with the LMIs criteria.

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