

IMPACT OF TEMPERATURE CHANGE ON THE POPULATION DYNAMICS OF THE MAIZE PEST *BUSSEOLA FUSCA*

J. P. NTAHOMVUKIYE^{1,4}, G. KOLAYE^{2,4,*}, S. BOWONG^{3,4} AND J. KURTHS^{5,6}

Abstract. We provided in this work a theoretical framework to study and to simulate the population dynamics of *Busseola fusca* (*B. fusca*): maize pest. The aim is to simulate and predict the presence levels of *Busseola fusca* under the influence of temperature variations and control actions. Based on the life cycle of *B. fusca*, we first propose a mathematical model to study the population dynamics of this maize pest. Some parameters are taken to be temperature-dependent. This led to a system of non-autonomous differential equations. Also, the classical control strategies are incorporated in the model. We present the theoretical analysis of the model. For the model with constant parameters, we compute the basic offspring number \mathcal{N}_0 that determines the evolution of the population of this insect and establish that the trivial equilibrium is globally asymptotically stable whenever $\mathcal{N}_0 < 1$, while if $\mathcal{N}_0 > 1$, the non trivial equilibrium is globally asymptotically stable. For the model with temperature variations, we find two explicit thresholds parameters \mathcal{N}_{max} and \mathcal{N}_{min} that bound the basic offspring number \mathcal{N}_0 (such that $\mathcal{N}_{max} \leq \mathcal{N}_0 \leq \mathcal{N}_{min}$), and use them to prove the extinction and the persistence of the pest within a maize field. We prove analytically and by numerical simulations that *B. fusca* persists uniformly within a maize field when $\mathcal{N}_{min} > 1$ and tends to disappears within a maize field when $\mathcal{N}_{max} < 1$. The theoretical results are illustrated by numerical simulations. They suggest further that spraying insecticides to kill larvae and destroying residues after harvest significantly reduce the population *Busseola fusca* more than other control actions.

Mathematics Subject Classification. 92D40, 92D45, 34D20, 37N25.

Received August 21, 2022. Accepted January 9, 2025.

1. INTRODUCTION

Maize is the most important cereal crop in Africa and is the food for millions of people, especially in sub-Saharan Africa. Maize farming within the African context is mostly done on small plots of land with low

Keywords and phrases: *Busseola fusca*, *Zea mays*, mathematical models, climatic factors, stability.

¹ Department of Mathematics, Faculty of Science, University of Burundi, PO Box 2700 Bujumbura, Burundi.

² Department of Mathematics and Computer Science, Faculty of Science, The University of Maroua, PO Box 814, Maroua, Cameroon.

³ Laboratory of Mathematics, Department of Mathematics and Computer Science Faculty of Science, University of Douala, PO Box 24157 Douala, Cameroon.

⁴ IRD, Sorbonne University, UMMISCO, 93143 Bondy France.

⁵ Potsdam Institute for Climate Impact Research (PIK), Telegraphenberg A 31, 14412 Potsdam, Germany.

⁶ Department of Physics, Humboldt Universitat zu Berlin, 12489 Berlin, Germany.

* Corresponding author: kolayegg@gmail.com

productivity. Increasing the productivity of these small-scale farming systems would ensure both food security in various contexts and an increased contribution to the economies of African countries [1].

In Africa, maize is used for both the feeding of animals and humans, who consume in the form of ears or grilled cooked or as flour. It is a cereal that is easy to store and transform. Maize is usually grown by the small farmer for local consumption, and yields tend to be very low, on average less than half those in Asia and America.

The production yield of these crops among peasants with limited financial resources is low, averaging 1 to 1.5 tons per hectare (ICIPE 2004). This low cereal crop yield is largely related to the constraints caused by crop pests, among which are the Lepidoptera stem borers. *Busseola fusca* (Fuller) (Lepidoptera, Noctuidae) is a major pest of maize and cultivated sorghum in sub-Saharan Africa. It is, on the African continent, one of the most important pests on the economic plan of some cultivated grasses [2]. Its larvae mainly drill stalks of maize and cultivated sorghum. *B. fusca* is endemic in sub-Saharan Africa where it has a wide geographical distribution. This pest is potentially destructive at the larval stage because by creating tunnels in the stems, it stops the photosynthesis and can cause the death of the plant. Feeding and training tunnels in the stem of maize by *B. fusca* larvae can destroy the growing point (dead center). Once inside the cereal stalks, the larvae feed on the marrow of the stem and then expel out of the stem large amount of fine dust corresponding to faeces and food waste. The development time of the larvae inside the stems varies between 26 and 33 days [3]. The damage leads to a general weakening of the plant, which, in the event of heavy rainfall, is then subjected to a high rate of lodging or breakage. In some cases, this damage can directly affect young ears of corn by destroying their axis bearing grain [4]. As *B. fusca* larvae are cannibal, only one older larva per corn stalk can be found.

The life cycle of *B. fusca* from egg to adult is completed within 7 to 8 weeks, that is, about 60 days, but this time varies with climatic conditions [5]. Temperature plays an important role in the development of *B. fusca*. The duration of the egg incubation period is influenced by temperature.

Due to the development time and survival, the optimal temperatures most favorable to the development of *B. fusca* are between 26 C and 30 C [6, 7].

The dynamics of *B. fusca* is complex due to the multiple interactions of this pest with plants as well as its bisexual and unisexual reproductive biology, genetics and its microscopic size (Brooks, 2017; Kaplan and Opperman, 1993). A deep understanding of the dynamics of the population of *R. similis* would have a significant impact on the effective prevention and management strategies (Brooks, 2017; Kaplan and Opperman, 1993). Mathematical modeling and numerical simulations have the potential, and offer a promising way, to achieve this. No effort has been devoted to the modeling and controlling of the population of this insect.

No effort has been made to model the population of this pest. But it is known that mathematical modeling now plays a key role in policy making, including health-economic or ecology-economic aspects; emergency planning and risk assessment; control-program evaluation; and monitoring of surveillance data. In research, it is essential in study design, analysis (including parameter estimation) and interpretation [8].

In this work, we study the dynamics of the population of *B. fusca* under the influence of climatic factors and control actions within a maize field. The aim is to provide a framework in which dynamic of population of *B. fusca* is investigated. There is no a mathematical model in literature which characterizes the dynamics of this population under the influence of temperature variations. Based on the general knowledge of the biology and ecology of this pest, we first formulate and analyze a mathematical model for population dynamics of *B. fusca* within a maize field. We present the theoretical analysis of the model. More precisely, we study the basic properties of the model such as the positivity of solutions and the boundedness of trajectories. We compute the trivial equilibrium point and derive the basic offspring number \mathcal{N}_0 that determines the extinction and the persistence of the pest within *Z. mays*. We compute equilibria and study their stability. We establish that the trivial equilibrium point is globally asymptotically stable when $\mathcal{N}_0 < 1$, while when $\mathcal{N}_0 > 1$, the trivial equilibrium point is unstable and there exists a unique a non-trivial equilibrium point which is globally stable asymptotically. The sensitivity analysis of the model is carried out to show the most important parameters of the model. We found that the parameter representing the eggs hatching rate influences more state variables of the system. This mean that, a control action reducing eggs hatching rate on the population of *B. fusca* should have a real impact on the dynamic of population. After, we extend the proposed model by taking into account

the the impact of temperature variation on the population dynamics of *B. fusca*. The control actions based on existing biological and chemical methods have been integrated in the model in order to reduce the population of *B. fusca* a maize field. We present the qualitative analysis of the model. We compute equilibria and study their stabilities. The stability of equilibria was investigated using the theory of periodic cooperative systems with a concave non-linearity. We show that there exist two explicit thresholds parameters \mathcal{N}_{max} and \mathcal{N}_{min} such that $\mathcal{N}_{max} \leq \mathcal{N}_0 \leq \mathcal{N}_{min}$, and use them to prove the extinction and the persistence of the pest within a maize field. More precisely, we prove that the population of *B. fusca* tends to disappear within a maize field when $\mathcal{N}_{max} < 1$ and persists uniformly within a maize field when $\mathcal{N}_{min} > 1$. Numerical simulations are presented to support the theory and to get insight on the role of temperature variations and the impact of chemical and biological controls on the dynamics of the population of *B. fusca*. Through numerical simulations, we found that the control action on the parameter β is the best strategy among the existing methods. This control corresponds to the spraying insecticides to kill the larvae or destroy the crop residues to prevent the larvae from emerging as butterflies. The numerical simulations reveal also that there is no real difference when the control action are implemented on the 1st month, 2nd month or 3rd month during the cropping period.

The rest of the paper is organized as follows. After mathematical formulation of model in Section 2 we provided in Section 3 a thorough mathematical analysis of model with numerical simulations. After, this model was extended by considering the effects of temperature variation and chemical and biological controls in Section 4. We present a qualitative analysis of the model obtained; we give the conditions of extinction and persistence of the pest and numerical simulations are presented to confirm theoretical results and to evaluate impacts of the controls strategies. We conclude the paper in Section 5.

2. MODEL CONSTRUCTION

Considering that temperature plays an important role in the development [6, 7] of *B. fusca* and that the length of the egg incubation period is influenced by temperature, we propose a model describing the evolution of *B. fusca* as a function of temperature in the region.

To construct the model, we divide the *B. fusca* population into 3 compartments: egg compartment E , larval compartment L and adult compartment A . *B. fusca* is a major pest, especially on maize, throughout Africa south of the Sahara [9]. This mean that maize is main feeding resource of the larvae and we denote by R the amount of this resource useful for the growth and survival of *B. fusca* larvae.

It is important to notice that the larvae stage of *B. fusca* is main threat of maize growth. The Adults of *B. fusca* do not feed on plant parts of the corn plant. After emergence, adult moths feed on nectar from flowers, mainly during their reproductive phase, and not on corn leaves or other parts of the host plant [10].

The structure of the model is depicted through the Figure 1 and the meanings of the model parameters in Table 1. Since egg hatch, emergence larval to adult and their natural mortality respective are generally related to temperature variation [6, 7], we will express the parameters γ , β , μ_E et μ_L in terms of temperatures.

We denote by $\gamma(T)$, $\beta(T)$, $\mu_E(T)$ et $\mu_L(T)$ the temperature-dependent parameters that represents respectively the rate of eggs hatching into larvae, the rate of emergence larval to adult and their natural mortality rate respective. These parameters are dependent of temperature variation T .

We assume that the adults lay eggs in the proportion $\alpha(1 - \frac{E}{K})$, where α is the average number of eggs laid and K is the limit capacity.

The term $\epsilon\eta RL$ represents the consumption of the resource by the larvae. This consumption maintains the survival of the larvae, *i.e.* consumption reduces the natural mortality of the larvae. Consequently, the expression $\mu_L(T) - \epsilon\eta R$ remains positive for all times $t > 0$.

Using the flowchart in Figure 1, the dynamics of *B. fusca* population under the temperature variations can be described by the following system of differential equations:

$$\begin{cases} \dot{E} = \alpha(1 - \frac{E}{K})A - (\mu_E(T) + \gamma(T)) E, \\ \dot{L} = \gamma(T)E - (\beta(T) + \mu_L(T) - \epsilon\eta R + dL) L, \\ \dot{A} = \beta(T)L - \mu_A A, \\ \dot{R} = \Lambda - \eta RL - \delta R. \end{cases} \quad (2.1)$$

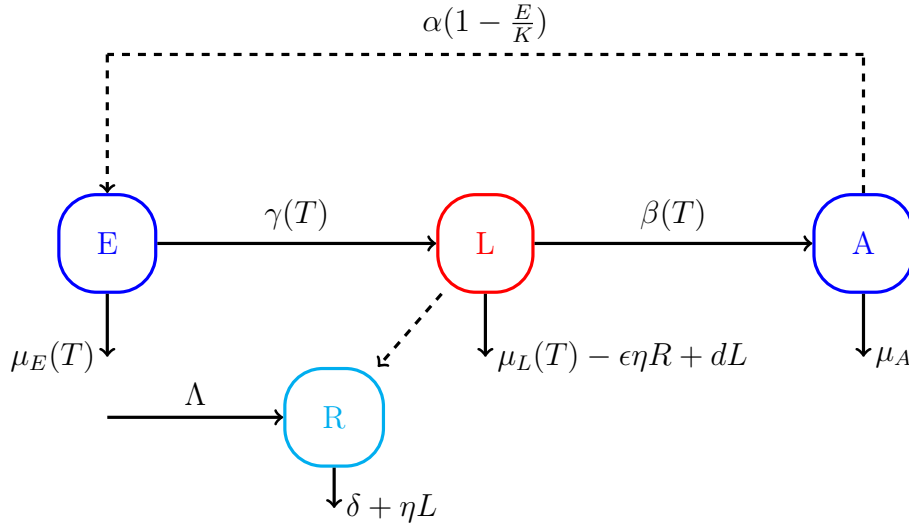


FIGURE 1. Flowchart of the model.

TABLE 1. Parameters description of system (2.1).

Parameter	Description	Units
α	Average number of eggs laid by adults females	day ⁻¹
$\gamma(T)$	Eggs hatching rate	day ⁻¹
$\beta(T)$	Emergence rate from larvae to adults	day ⁻¹
K	Limit capacity	number
$\mu_E(T)$	Eggs mortality rate	day ⁻¹
$\mu_L(T)$	Larvae mortality rate	day ⁻¹
d	Larvae competition rate	day ⁻¹ number ⁻¹
μ_A	Mortality rate of adults	day ⁻¹
Λ	Recruitment rate of the plant biomass	gr. day ⁻¹
δ	Loss rate of plant biomass	day ⁻¹
η	the consumption rate of the biomass of the plant	day ⁻¹
ϵ	Conversion rate of the plant biomass to larvae biomass	day ⁻¹

The parameters with units of system (2.1) are given in Table 1.

3. ANALYSIS OF THE MODEL WITH CONSTANT PARAMETER VALUES

Herein, we consider the case where all the parameters are constant. In this case, the parameters γ, β, μ_E and μ_L do not depend on the temperature and system (2.1) reduces to

$$\begin{cases} \dot{E} = \alpha(1 - \frac{E}{K})A - (\mu_E + \gamma) E, \\ \dot{L} = \gamma E - (\beta + \mu_L - \epsilon\eta R + dL) L, \\ \dot{A} = \beta L - \mu_A A, \\ \dot{R} = \Lambda - \eta RL - \delta R. \end{cases} \quad (3.1)$$

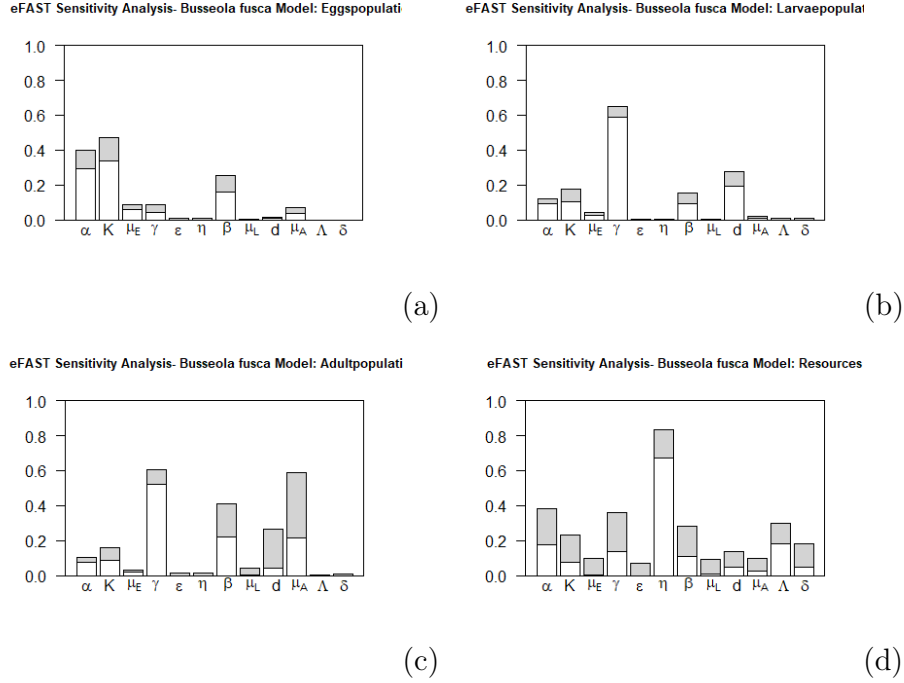


FIGURE 2. eFast sensitivity analysis.

3.1. Sensitivity analysis

In order to detect the most sensitive parameters, we perform here the sensitivity analysis of the model (3.1). This can help us to predict the effect of each parameter on the model results and classify them according to their degree of sensitivity. For this, we use the eFast sensitivity method which highlights the effects of the first order called main effects and the total effects that combine the main effects and all the interaction effects of the parameters on the outputs of the model [11]. It is a global sensitivity technique based on the decomposition or partitioning of the variance. The variance of the model output is decomposed into components resulting from the individual effects of parameters as well as their interactions [12, 13].

In Table 2, we provide ranges of values for the parameters of model (3.1).

Table 3 presents the parameters that are significantly correlated at least at one state variable of system (2.1).

According to Table 3, γ is the parameter that most influences the state variables of system (2.1). It is then a potential parameter for control actions. Controlling egg hatching rate should be the most effective control strategy.

3.2. Basic properties

Herein, we study the basic properties of system (2.1) which are essential for the proof of the stability results. System (3.1) can be written in the following compact form:

$$\dot{X} = A(X)X + B, \quad (3.2)$$

where $X = (E, L, A, R)^T$, $B = (0, 0, 0, \Lambda)^T$ and

$$A(X) = \begin{pmatrix} -(\mu_E + \gamma + \frac{\alpha}{K}A) & 0 & \alpha & 0 \\ \gamma & -(\beta + \mu_L - \epsilon\eta R + dL) & 0 & 0 \\ 0 & \beta & -\mu_A & 0 \\ 0 & 0 & 0 & -(\delta + \eta L) \end{pmatrix}.$$

TABLE 2. Range of values for the parameters of system (3.1)

Parameters	Value	Range	Source
α	25 number day ⁻¹	[10–150]	[14]
K	1000 number	[100–10 000]	[14]
μ_E	0.5 day ⁻¹	[0.001–0.999]	Estimated
γ	Variable	[0.001–0.999]	[14]
β	0.25 day ⁻¹	[0.001–0.999]	[14]
μ_L	0.35 day ⁻¹	[0.001–0.999]	[2, 6]
ϵ	0.05	[0.001–0.999]	Assumed
η	0.1 day ⁻¹	[0.001–0.999]	Assumed
d	0.45 day ⁻¹	[0.001–0.999]	Assumed
μ_A	0.55 day ⁻¹	[0.001–0.999]	Assumed
Λ	3 gr.day ⁻¹	[1–10]	Assumed
δ	0.15 day ⁻¹	[0.001–0.999]	Assumed

TABLE 3. List of parameters more influencing of system (2.1).

State variables	E	L	A	R
Correlated parameters	K	γ	γ, μ_A	η

The right hand side of system (3.2) is continuous and indefinitely differentiable on \mathbb{R}_+^4 . It is therefore locally Lipschitz on \mathbb{R}_+^4 , which guarantees the existence and uniqueness of the solution of system (3.2) associated with the initial conditions $(t_0, X(t_0)) \in \mathbb{R}_+ \times \mathbb{R}_+^4$.

Now, we study the positivity of dynamical system thereby guaranteeing that system (3.2) leaves the positive orthant invariant \mathbb{R}_+^4 . To guarantee the invariance on the positive orthant \mathbb{R}_+^4 , it is necessary and sufficient to verify that the boundaries of \mathbb{R}_+^4 are impassable by the system trajectories initialized in \mathbb{R}_+^4 . Therefore, it must be ensured that for all i , if the state variable X_i is zero at time t (with $X(t) \geq 0$), \dot{X}_i is positive, which will ensure that for all $T > t$, $X_i(T) \geq 0$ and ensure the positivity of the system.

Another way to prove positivity is to show that the matrix of the system is a Metzler matrix

Definition 3.1. A Metzler matrix is a matrix with off-diagonal entries nonnegative [15, 16]

We recall the Theorem 1.7, page 22 of [17].

Proposition 3.2. Consider the following system:

$$\dot{X} = A(X)X + B. \quad (3.3)$$

If $A(X)$ is Metzler matrix for all $X \in \mathbb{R}_+^4$, and $B \geq 0$, then system (3.3) leaves positively invariant the positive orthant \mathbb{R}_+^4 .

Proof. To prove the positivity of system (3.2), it is necessary to show that, $\forall X_0 \in \mathbb{R}_+^4, \forall i = 1, 2, \dots, 4, \forall t \geq 0$ such that $X_i = 0$, then $\dot{X}_i \geq 0$. Thus, $\forall i, X_i = 0$ implies

$$\begin{aligned} \dot{X}_i &= a_{ii}X_i + \sum_{j \neq i} a_{ij}X_j + b_i \\ &= \sum_{j \neq i} a_{ij}X_j + b_i. \end{aligned}$$

To ensure that $\dot{X}_i \geq 0$ for all the positive values of X_j , it is necessary that $\forall i, b_i \geq 0$ and $\forall i, i \neq j, a_{ij} \geq 0$. Therefore, a necessary condition for system (3.2) to be positive is that $A(X)$ is a Metzler matrix and $B \geq 0$. This achieves the proof. \square

For system (3.2), the matrix $A(X)$ is a Metzler matrix and $B > 0$. This guarantees the positivity of solutions of system (3.2).

Since the larvae consume the resource for their survival, we assume in the sequel that the term $s = \mu_L - \epsilon\eta\frac{\Lambda}{\delta}$ which represents the reduction in the natural mortality of the larvae due to the consumption of the resource is positive.

Proposition 3.3. *The set*

$$\Omega = \left\{ (E, L, A, R) \in \mathbb{R}_+^4; \quad E \leq K, \quad L \leq \frac{\gamma K}{\beta + s}, \quad A \leq \frac{\beta\gamma K}{\mu_A(\beta + s)}, \quad R \leq \frac{\Lambda}{\delta} \right\} \quad (3.4)$$

is a positively invariant set for system (3.1)

Proof. From the first equation, we show that if $E(0) \leq K$, then $\forall t > 0, E(t) \leq K$. Suppose that there exists $t_0 > 0$ such that $E(t_0) > K$. By continuity, there exists $t_1 < t_0$ such that

$$E(t_1) = K \text{ and } \forall t \in]t_1, t_0], \text{ we have } E(t) > K.$$

Thus, if $E(t_1) = K$, the second equation of system (3.1) becomes

$$\dot{E}(t_1) = -(\mu_E + \gamma)K < 0.$$

Therefore, there exists $t^* \in]t_1, t_0[$ such that $E(t^*) < K$, which is absurd because one has that $E(t) > K, \forall t \in]t_1, t_0[$. Thus, for all $t > 0$, we have $E(t) \leq K$ if $E(0) \leq K$.

Consider now the last equation of system (3.1):

$$\dot{R}(t) = \Lambda - \eta R(t)L(t) - \delta R(t).$$

Since the variables of system (3.1) are all positive, one has that

$$\dot{R}(t) \leq \Lambda - \delta R(t).$$

Using Gronwall's inequality gives

$$R(t) \leq \frac{\Lambda}{\delta} + \left(R(0) - \frac{\Lambda}{\delta} \right) e^{-\delta t}.$$

It then follows that $R(t) \leq \frac{\Lambda}{\delta}$ for all $t \geq 0$ if $R(0) \leq \frac{\Lambda}{\delta}$.

From the second equation of system, one has that

$$\dot{L}(t) = \gamma E(t) - (\beta + \mu_L - \epsilon\eta R(t) + dL(t)) L(t).$$

Using the fact that $R(t) \leq \frac{\Lambda}{\delta}$, it comes that

$$\dot{L}(t) \leq \gamma K - (\beta + s) L(t).$$

By Gronwall's inequality, one has that

$$L(t) \leq \frac{\gamma K}{\beta + s} + \left(L(0) - \frac{\gamma K}{\beta + s} \right) e^{-(\beta+s)t}.$$

Hence, it follows that $L(t) \leq \frac{\gamma K}{\beta+s}$ for all $t \geq 0$, if $L(0) \leq \frac{\gamma K}{\beta+s}$.

Finally, from the third equation of system (3.1), one has

$$\dot{A}(t) = \beta L(t) - \mu_A A(t).$$

Using the fact that $L(t) \leq \frac{\gamma K}{\beta+s}$, the above equation becomes

$$\dot{A}(t) \leq \beta \frac{\gamma K}{\beta + s} - \mu_A A(t).$$

Using the Gronwall's inequality, again yields

$$A(t) \leq \frac{\beta \gamma K}{\mu_A (\beta + s)} + \left(A(0) - \frac{\beta \gamma K}{\mu_A (\beta + s)} \right) e^{-\mu_A t}.$$

This implies that $A(t) \leq \frac{\beta \gamma K}{\mu_A (\beta+s)}$, for all $t \geq 0$ if $A(0) \leq \frac{\beta \gamma K}{\mu_A (\beta+s)}$. Therefore, the set Ω is a positively invariant set for system (3.1). This achieves the proof. □

3.2.1. Basic offspring number and local stability of the trivial equilibrium point

Herein, we compute the trivial equilibrium point and derive the basic offspring number \mathcal{N}_0 . Indeed, the evolution of a population is generally linked to its number of basic descendants. This is the average number of females of *Busseola fusca* produced by one female of *Busseola fusca* over its lifespan.

we use the method of Van den Driessche and J. Watmough [18] for the computation of the basic offspring number.

System (3.1) has a trivial equilibrium point given by

$$Q_0 = \left(0, 0, 0, \frac{\Lambda}{\delta} \right). \quad (3.5)$$

System (3.1) can be written as follows:

$$\dot{X} = \mathcal{R}(X) - \mathcal{T}(X),$$

where

$$\mathcal{R}(X) = \begin{bmatrix} \alpha \left(1 - \frac{E}{K} \right) A \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathcal{T}(X) = \begin{bmatrix} (\mu_E + \gamma) E \\ (\beta + \mu_L - \epsilon \eta R + dL) L - \gamma E \\ \mu_A A - \beta L \end{bmatrix}.$$

The Jacobian matrices of $\mathcal{R}(X)$ and $\mathcal{T}(X)$ at the trivial equilibrium point Q_0 are respectively,

$$R = J_{\mathcal{R}}(Q_0) = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad T = J_{\mathcal{T}}(Q_0) = \begin{bmatrix} (\mu_E + \gamma) & 0 & 0 \\ -\gamma & \beta + s & 0 \\ 0 & -\beta & \mu_A \end{bmatrix}$$

According to Van den Driessche and Watmough [18] and Diekmann *et al.* [19], the matrix of the next generation is the product of the matrix R and the inverse matrix of T , that is

$$RT^{-1} = \begin{bmatrix} \frac{\alpha\beta\gamma}{\mu_A(\beta+s)(\mu_E+\gamma)} & \frac{\alpha\beta}{\mu_A(\beta+s)} & \frac{\alpha}{\mu_A} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

The basic offspring number is the spectral radius of the next-generation matrix given by

$$\mathcal{N}_0 = \rho(RT^{-1}) = \frac{\alpha\beta\gamma}{\mu_A(\beta+s)(\mu_E+\gamma)}. \quad (3.6)$$

The basic offspring number is calculated from the stability of the trivial equilibrium point Q_0 . This number measures the degree of replacement of one generation by the next. It expresses the average number of young females born from a female who would be subjected each age, during her fertile life, to the fertility and mortality conditions observed during the period considered. This number reflects the fact that a number of females die before the end of their fertility.

We have the following result about the local stability of the trivial equilibrium point

Lemma 3.4. *The trivial equilibrium Q_0 of system (3.1) is locally asymptotically stable if $\mathcal{N}_0 \leq 1$ and unstable if $\mathcal{N}_0 > 1$.*

The ecological implication of Lemma 3.4 is that a sufficiently small flow of female of *Busseola fusca* in a maize will not generate outbreak of infection unless $\mathcal{N}_0 \geq 1$. For a better control of this pest, the global asymptotic stability (GAS) of Q_0 is needed.

3.2.2. Global stability of the trivial equilibrium point

Here, we study the global stability of the trivial equilibrium point of system (3.1). To do so, we will use the comparison criterion to study the stability by considering the following system in which the fourth equation was decoupled from system (3.1):

$$\begin{cases} \dot{E} = \alpha(1 - \frac{E}{K})A - (\mu_E + \gamma)E, \\ \dot{L} = \gamma E - (\beta + \mu_L - \epsilon\eta R + dL)L, \\ \dot{A} = \beta L - \mu_A A. \end{cases} \quad (3.7)$$

Since $R(t) \leq \frac{A}{\delta}$ for all $t \geq 0$, one has that $\epsilon\eta RL \leq \epsilon\eta \frac{A}{\delta} L$. With this in mind, system (3.7) becomes

$$\begin{cases} \dot{E} \leq \alpha A - (\mu_E + \gamma)E, \\ \dot{L} \leq \gamma E - (\beta + s)L, \\ \dot{A} \leq \beta L - \mu_A A. \end{cases} \quad (3.8)$$

Now, consider the following linear auxiliary system:

$$\begin{cases} \dot{\bar{E}} = \alpha\bar{A} - (\mu_E + \gamma)\bar{E}, \\ \dot{\bar{L}} = \gamma\bar{E} - (\beta + s)\bar{L}, \\ \dot{\bar{A}} = \beta\bar{L} - \mu_A\bar{A}. \end{cases} \quad (3.9)$$

System (3.9) can be written as follows:

$$\dot{\bar{X}} = \mathcal{R}(\bar{X}) - \mathcal{T}(\bar{X}),$$

where

$$\mathcal{R}(\bar{X}) = \begin{bmatrix} \alpha\bar{A} \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathcal{T}(\bar{X}) = \begin{bmatrix} (\mu_E + \gamma)\bar{E} \\ (\beta + s)\bar{L} - \gamma\bar{E} \\ \mu_A\bar{A} - \beta\bar{L} \end{bmatrix}.$$

The Jacobian matrices of $\mathcal{R}(\bar{X})$ and $\mathcal{T}(\bar{X})$ at the trivial equilibrium point Q_0 are respectively,

$$\bar{R} = J_{\mathcal{R}}(Q_0) = \begin{bmatrix} 0 & 0 & \alpha \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \bar{T} = J_{\mathcal{T}}(Q_0) = \begin{bmatrix} (\mu_E + \gamma) & 0 & 0 \\ -\gamma & \beta + s & 0 \\ 0 & -\beta & \mu_A \end{bmatrix}.$$

The basic offspring number is the spectral radius of the next-generation matrix $\bar{R}\bar{T}^{-1} = \bar{\mathcal{N}}_0$.

We have proved the following result.

Lemma 3.5. *The trivial equilibrium point Q_0 of system (3.1) is globally asymptotically stable if $\mathcal{N}_0 \leq 1$ in Ω .*

3.2.3. The non-trivial equilibrium point and its stability

Here, we compute the non-trivial equilibrium point and study its stability.

Let $Q^* = (E^*, L^*, A^*, R^*)$ be the non-trivial equilibrium of system (3.1) with $E^* \neq 0$, $L^* \neq 0$, $A^* \neq 0$ and $R^* \neq 0$ satisfying the following system of equations:

$$\begin{cases} \alpha(1 - \frac{E^*}{K})A^* - (\mu_E + \gamma)E^* = 0, \\ \gamma E^* - (\beta + \mu_L - \epsilon\eta R^* + dL^*)L^* = 0, \\ \beta L^* - \mu_A A^* = 0, \\ \Lambda - \eta R^* L^* - \delta R^* = 0. \end{cases} \quad (3.10)$$

Solving the first, second and third equations of system (3.10) in term of L^* gives

$$A^* = \frac{\beta}{\mu_A}L^*, \quad E^* = \frac{\alpha\beta KL^*}{K\mu_A(\mu_E + \gamma) + \alpha\beta L^*} \quad \text{and} \quad R^* = \frac{m_1 - m_5 + (m_2 + m_3)L^* + m_4L^{*2}}{m_6 + m_7L^*}, \quad (3.11)$$

where

$$\begin{aligned} m_1 &= K\mu_A(\mu_E + \gamma)(\beta + \mu_L), & m_2 &= \alpha\beta(\beta + \mu_L), & m_3 &= K\mu_A(\mu_E + \gamma)d, \\ m_4 &= \alpha\beta d, & m_5 &= K\alpha\gamma\beta, & m_6 &= \epsilon\eta K\mu_A(\mu_E + \gamma) \quad \text{and} \quad m_7 = \epsilon\eta\alpha\beta. \end{aligned}$$

TABLE 4. Number of positive solution of the polynomial (3.12).

Condition for \mathcal{N}_0	a_3	a_2	a_1	a_0	Positive solutions
$\mathcal{N}_0 < 1$	-	-	-	-	No solution
$\mathcal{N}_0 > 1$	-	-	-	+	One solution
$\mathcal{N}_0 > 1$	-	-	+	+	One solution

Now, plugging the expression of R^* in the last equation of system yields the following third order polynomial:

$$a_3 L^{*3} + a_2 L^{*2} + a_1 L^* + a_0 = 0, \quad (3.12)$$

where

$$\begin{aligned} a_3 &= -\eta m_4 = -\eta \alpha \beta d < 0; \\ a_2 &= -(\eta(m_2 + m_3) + \delta m_4) \\ &= -(\eta \alpha \beta (\beta + \mu_L) + \eta K \mu_A (\mu_E + \gamma) d + \delta \alpha \beta d) < 0; \\ a_1 &= m_7 \Lambda - \eta m_1 + \eta m_5 - \delta(m_2 + m_3) \\ &= \epsilon \eta \Lambda \alpha \beta - \eta K \mu_A (\mu_E + \gamma) (\beta + \mu_L) + \eta K \alpha \gamma \beta \\ &\quad - \delta (\alpha \beta (\beta + \mu_L) + K \mu_A (\mu_E + \gamma) d); \\ &= -\alpha \beta \delta (\beta + s) \\ &\quad - \eta K \mu_A (\mu_E + \gamma) (\beta + \mu_L) \left(1 - \frac{\alpha \gamma \beta}{\mu_A (\mu_E + \gamma) (\beta + \mu_L)} \right) \\ &\quad - \delta d K \mu_A (\mu_E + \gamma); \\ a_0 &= m_6 \Lambda - \delta m_1 + \delta m_5; \\ &= -K \mu_A (\mu_E + \gamma) ((\beta + s) (1 - \mathcal{N}_0)), \end{aligned}$$

with \mathcal{N}_0 defined as in equation (3.6). Thus, positive non trivial equilibria Q^* are obtained by solving equation (3.12) and substituting the result (positive values of L^*) into the expressions of the variables of system (3.10) at the steady state. Clearly, a_0 is negative or positive depending whether \mathcal{N}_0 is less than or greater than unity, respectively. Thus, the number of possible real roots of the polynomial equation (3.12) depends on the signs of a_3 , a_2 , a_1 and a_0 . This can be analyzed using the Descartes Rule of Signs on the quadratic polynomial $f(L^*) = a_3 L^{*3} + a_2 L^{*2} + a_1 L^* + a_0$. The various possibilities for the roots of equation (3.12) are summarized in Table 4.

From Table 2, it is clearly appears that there is only one non-trivial equilibrium point when $\mathcal{N}_0 > 1$ and no non-trivial equilibrium point if $\mathcal{N}_0 \leq 1$. Thus, we have proved the following result.

Lemma 3.6. *System (3.1) has only one non-trivial equilibrium point $Q^* = (E^*, L^*, A^*, R^*)$ where E^* , L^* , A^* and R^* are defined in equations (3.11) when $\mathcal{N}_0 > 1$.*

Now, using the center manifold theory, we are going to study the stability of the non-trivial equilibrium. For this, we use the theory of the central variety [20], as described in the theorem A.1 of Castillo-Chavez and Song [21].

We have the following result.

Theorem 3.7. *System (3.1) has exactly one non-trivial equilibrium that is locally asymptotically stable if $\mathcal{N}_0 > 1$ but close to 1.*

Proof. We establish the local stability of the non-trivial equilibrium point of system (3.1). To do so, the following simplification and change of variables are made first to all. Let

$$E = x_1, \quad L = x_2, \quad A = x_3, \quad R = x_4.$$

With this in mind, using the vector $x = (x_1, x_2, x_3, x_4)$, system (3.1) can be written in the form $\dot{x} = f(x)$ with $f = (f_1, f_2, f_3, f_4)$ as follows

$$\begin{cases} \dot{x}_1 = \alpha(1 - \frac{x_1}{K})x_3 - (\mu_E + \gamma)x_1, \\ \dot{x}_2 = \gamma x_1 - (\beta + \mu_L - \epsilon\eta x_4 + dx_2)x_2, \\ \dot{x}_3 = \beta x_2 - \mu_A x_3, \\ \dot{x}_4 = \Lambda - \eta x_4 x_2 - \delta x_4. \end{cases} \quad (3.13)$$

System (3.13) has a trivial equilibrium point $Q_0 = (0, 0, 0, \frac{\Lambda}{\delta})$. The Jacobian of system (3.13) calculated at the trivial equilibrium point Q_0 is

$$J(Q_0) = \begin{pmatrix} -(\mu_E + \gamma) & 0 & \alpha & 0 \\ \gamma & -(\beta + s) & 0 & 0 \\ 0 & \beta & -\mu_A & 0 \\ 0 & -\eta\frac{\Lambda}{\delta} & 0 & -\delta \end{pmatrix}.$$

The basic offspring number of the transformed system (3.13) is the same as that the original system given by (3.1). Therefore, choosing α as bifurcation parameter by solving the equation $\mathcal{N}_0 = 1$, one obtains

$$\alpha = \alpha^* = \frac{\mu_A(\mu_E + \gamma)(\beta + s)}{\beta\gamma}. \quad (3.14)$$

It follows that the Jacobian $J(Q_0)$ of system (3.13) at Q_0 , with $\alpha = \alpha^*$, denoted by $J_{\alpha^*}(Q_0)$ has a simple zero eigenvalue (with all other eigenvalues having negative real parts). Hence, the Center Manifold theory [20] can be used to analyze the dynamics of system (3.13). In particular, the theorem of Castillo and Song [21] will be used to show that when $\mathcal{N}_0 > 1$, there exists an endemic equilibrium of system (3.13) which is locally asymptotically stable for $\mathcal{N}_0 > 1$ near 1 under certain conditions. In order to apply the above theorem, it should be noted that we used α^* as the bifurcation parameter, in place of Φ given in Theorem A.1

To apply the theorem A.1 of Castillo-Chavez and Song, one must first perform the calculations.

Eigenvectors of $J_{\alpha^*}(Q_0)$:

For $\mathcal{N}_0 = 1$, it can be shown that the Jacobian matrix of system (3.13) has a right eigenvector w corresponding to the zero eigenvalue), given by

$$w = (w_1, w_2, w_3, w_4) = \left(\beta + s, \gamma, \frac{\beta\gamma}{\mu_A}, -\frac{\eta\Lambda\gamma}{\delta^2} \right),$$

and a left eigenvector v satisfying $v \cdot w = 1$ given by $v = (v_1, v_2, v_3, v_4)$ where

$$\begin{aligned} v_1 &= \frac{\mu_A}{\mu_A(\mu_E + \gamma) + (\beta + s)(\mu_A + \mu_E + \gamma)}, \\ v_2 &= \frac{\mu_A(\mu_E + \gamma)}{\gamma\mu_A(\mu_E + \gamma) + \gamma(\beta + s)(\mu_A + \mu_E + \gamma)}, \\ v_3 &= \frac{\mu_A(\mu_E + \gamma)(\beta + s)}{\gamma\beta\mu_A(\mu_E + \gamma) + \gamma\beta(\beta + s)(\mu_A + \mu_E + \gamma)}, \\ v_4 &= 0. \end{aligned}$$

Computation of b: We calculate b in order to find its sign. To do this, we can show that the only non-zero second derivative of f applied to (Q_0, α^*) is given by

$$\frac{\partial^2 f_1}{\partial x_3 \partial \alpha^*} = 1.$$

It then follows that

$$\begin{aligned} b &= \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \Phi}, \\ &= v_1 w_3 \frac{\partial^2 f_1}{\partial x_3 \partial \alpha^*} \\ &= \frac{\gamma \beta}{\mu_A(\mu_E + \gamma) + (\beta + s)(\mu_A + \mu_E + \gamma)} > 0. \end{aligned}$$

Computation of a: We can show that the only non-zero partial derivatives are given by

$$\frac{\partial^2 f_1}{\partial x_1 \partial x_3} = \frac{\partial^2 f_1}{\partial x_3 \partial x_1} = -\frac{\alpha^*}{K}, \quad \frac{\partial^2 f_2}{\partial x_2^2} = -2d, \quad \frac{\partial^2 f_2}{\partial x_2 \partial x_4} = \frac{\partial^2 f_2}{\partial x_4 \partial x_2} = \epsilon \eta \quad \text{and} \quad \frac{\partial^2 f_4}{\partial x_2 \partial x_4} = \frac{\partial^2 f_4}{\partial x_4 \partial x_2} = -\eta.$$

It then follows that

$$\begin{aligned} a &= \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j} (0, 0) \\ &= 2v_1 w_1 w_3 \frac{\partial^2 f_1}{\partial x_1 \partial x_3} + v_2 w_2^2 \frac{\partial^2 f_2}{\partial x_2^2} + 2v_2 w_2 w_4 \frac{\partial^2 f_2}{\partial x_2 \partial x_4} + 2v_4 w_2 w_4 \frac{\partial^2 f_4}{\partial x_2 \partial x_4} \\ &= -\frac{\alpha^*}{K} v_1 w_1 w_3 - 2d v_2 w_2^2 + 2\epsilon \eta v_2 w_2 w_4 - 2\eta v_4 w_2 w_4 \\ &= -\frac{\alpha^*}{K} \frac{\gamma \beta (\beta + s)}{\mu_A(\mu_E + \gamma) + (\beta + s)(\mu_A + \mu_E + \gamma)} \\ &\quad - 2d \frac{\gamma \mu_A (\mu_E + \gamma)}{\mu_A(\mu_E + \gamma) + (\beta + s)(\mu_A + \mu_E + \gamma)} \\ &\quad - 2\epsilon \eta^2 \frac{\Lambda \gamma}{\delta^2} \frac{\mu_A (\mu_E + \gamma)}{\mu_A(\mu_E + \gamma) + (\beta + s)(\mu_A + \mu_E + \gamma)} \\ &< 0 \end{aligned}$$

Thus, $a < 0$ and $b > 0$. Then (by Thm. A.1, Item(4)) we have established the result on the local stability of the non-trivial equilibrium point Q^* of system (3.1) (note that this result holds for $\mathcal{N}_0 > 1$ but close to 1). This concludes the proof. \square

3.3. Numerical simulations

Herein, we present the results of numerical simulations in order to illustrate and validate the analytical results obtained. The parameter values used for numerical simulations are given in Table 2.

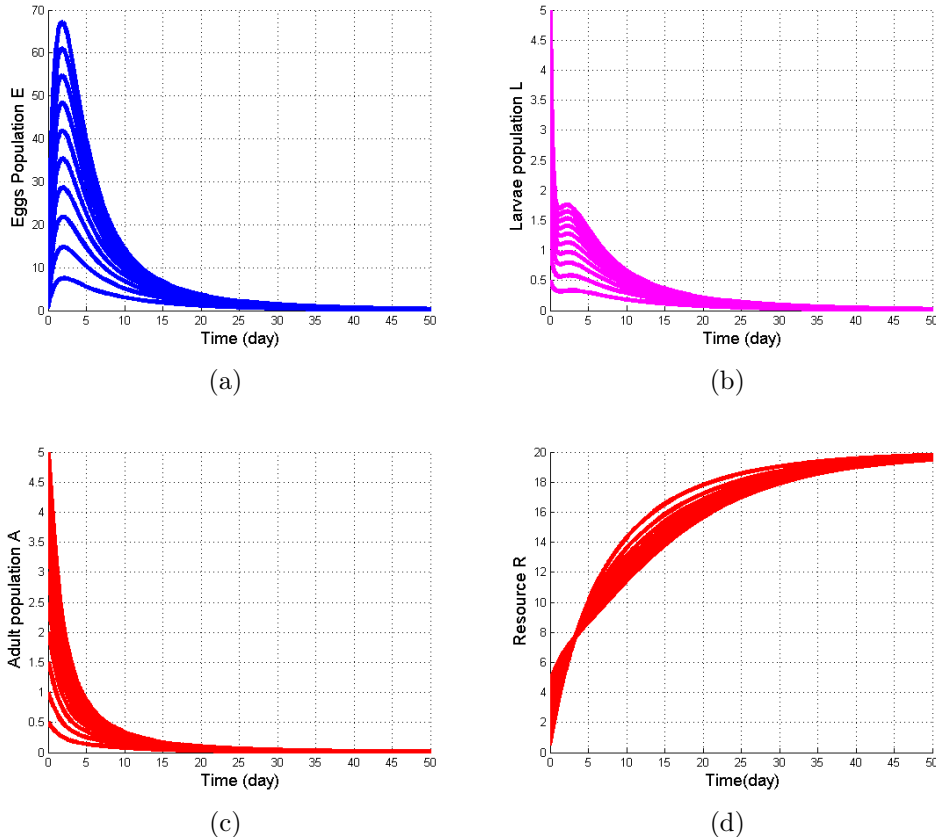


FIGURE 3. Simulation of system (3.1) when $\gamma = 0.08$ (such as $\mathcal{N}_0 = 0.9488$). (a) Eggs E ; (b) Larvae L , (c) Adult A and (d) Resources R . All other parameters are given in Table 2.

Figure 3 presents the trajectories of system (3.1) when $\gamma = 0.08$ (so that $\mathcal{N}_0 = 0.9488$). It illustrates that the trajectories of system (3.1) converge to the trivial equilibrium point Q_0 . This means that the population of *Busseola fusca* disappears in maize field.

Figure 4 shows the trajectories of system (3.1) when $\gamma = 0.2$ (such that $\mathcal{N}_0 = 2.0161$). It clearly appears that the trajectories of the model converge to the unique non-trivial equilibrium. This means that the population of *Busseola fusca* persists in the maize field.

4. ANALYSIS OF THE MODEL WITH TEMPERATURE DEPENDENT PARAMETERS

4.1. Modelling temperature-dependent parameters

A study by Glatz [6] shows that the temperature of 15 C is too cold for the fertility of *B. fusca* eggs. The number of eggs laid per female at 15 C is not lower than those laid at 20 C and 26 C but a high percentage of these eggs is sterile. Note also that the percentage of sterile eggs laid at a constant temperature of 30 C is 100% and that at this same temperature, the females have a short life. The temperature of 30 C is therefore too hot for the reproduction of *B. fusca*. It has been observed that the optimum temperature for oviposition and fertility of *B. fusca* eggs is between 20 C and 26 C.

Larval development is also conditioned by temperature. Usua [7] has observed that the temperature of 22 C stops the development of larvae, while temperatures of 32 C or higher kill larvae. In the Glatz study, *B. fusca* larvae completed their slowest cycle at 15 C and the fastest at 26 C. Given the mortality and slow development

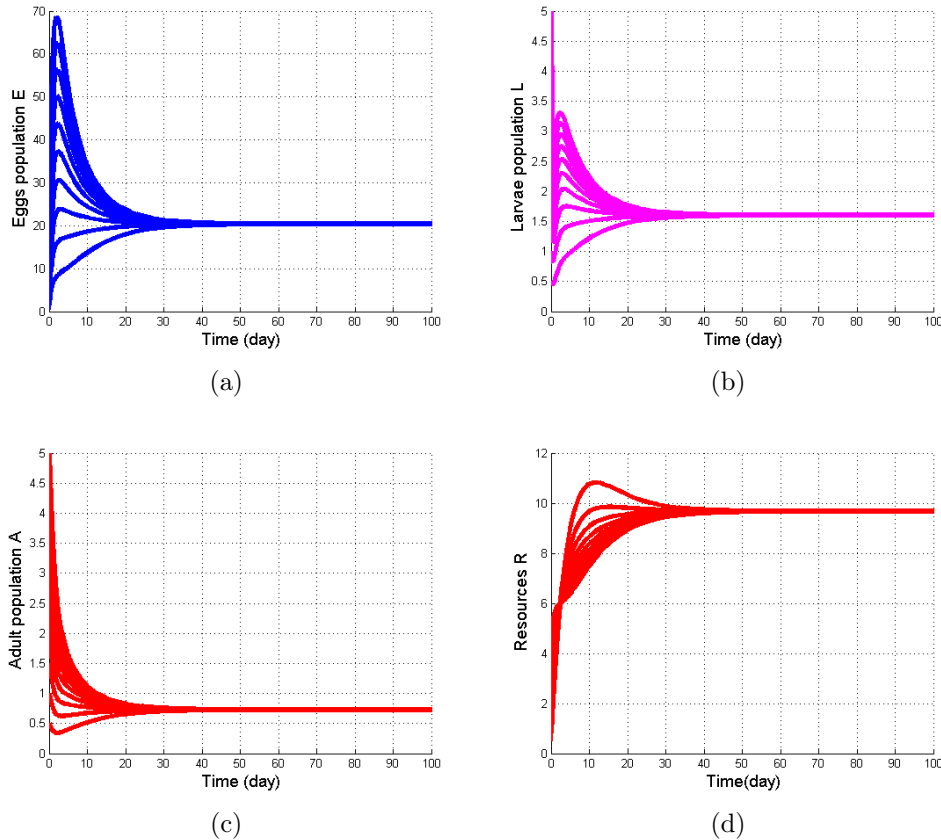


FIGURE 4. Simulation of system (3.1) when $\gamma = 0.2$ (such that $\mathcal{N}_0 = 2.0161$). (a) Eggs E ; (b) Larvae L , (c) Adult A and (d) Resources R . All other parameters are given in Table 2.

of larvae in temperatures between 15 C and 20 C, he concluded that low temperatures reduce the population of *B. fusca*.

To take into account the impact of climatic factors on the dynamics of *Busseola fusca*, we will integrate the fact that the evolution of the population of *Busseola fusca* is a function of the temperature of the environment.

Given that the eggs hatch and larval survival are generally related to temperature variation, we will modify system (3.1) by expressing the parameters γ , β , μ_E and μ_L in terms of temperature variations.

It is known that the hatching of egg is maximum when the temperature is favorable and decreases when the temperature deviates from the optimal temperature. Also, we assume that larval development is influenced by temperature. Then parameters γ and β reach the maximum value when the temperature is optimal and decreases when the temperature is unfavorable. Let $\psi \in \{\gamma, \beta\}$. We consider the following function of ψ to drive the value of γ and β in term of temperature variations:

$$\psi(T) = \psi_{min} + (\psi_{max} - \psi_{min}) \exp\left(-\left(\frac{T(t) - T_{opt-\psi}}{\mathcal{A}_T}\right)^2\right), \quad (4.1)$$

where ψ_{min} is the minimum value ψ , ψ_{max} the maximum value ψ , $T_{opt-\psi}$ is the optimal temperature and $\mathcal{A}_T = \frac{T_{max} - T_{min}}{2}$ is the amplitude of temperature variation.

The studies of Usua [7] had also shown that the mortality of eggs and larvae is really influenced by temperature. We also suppose that mortalities of eggs and larvae are minimum when the temperature is favorable

and decreases when the temperature deviates from the optimal temperature. Let $\mu \in \{\mu_E, \mu_L\}$. We consider the following function of μ to drive the value of μ_E and μ_L in term of temperature variations:

$$\mu(T) = \mu_{max} + (\mu_{min} - \mu_{max}) \exp\left(-\left(\frac{T(t) - T_{opt-\mu}}{\mathcal{A}_T}\right)^2\right), \quad (4.2)$$

where μ_{min} is the minimum mortality rate, μ_{max} the maximum mortality rate, $T_{opt-\mu}$ the optimal temperature and $\mathcal{A}_T = \frac{T_{max} - T_{min}}{2}$ the amplitude of temperature variations.

For every temperature value T , one has that

$$\underline{\gamma} \leq \gamma(T) \leq \bar{\gamma}, \quad \underline{\beta} \leq \beta(T) \leq \bar{\beta}, \quad \underline{\mu_E} \leq \mu_E(T) \leq \bar{\mu_E} \quad \text{and} \quad \underline{\mu_L} \leq \mu_L(T) \leq \bar{\mu_L}. \quad (4.3)$$

The control strategies for *Busseola fusca* shown are based on the following actions: spraying repellents such as neem to prevent adults from entering the crop and laying eggs, spraying contact or systemic insecticides to kill adults, spraying insecticides to kill larvae especially in the early larval stage and crop residues after harvest to prevent infection by adults emerging from pupae and to prevent them from laying eggs on new crops.

In order to take into account the control of the pest, some parameters are modified. The modification of these parameters corresponds to each control actions previously mentioned.

- (i) For the parameter μ_A , we have either contact or systemic insecticide spraying to kill the adults (butterflies);
- (ii) For the parameter α , spraying repellents to prevent the adults from laying eggs;
- (iii) To control the parameter β , we either spray contact or systemic insecticides to kill the larvae or destroy the crop residues to prevent the larvae from emerging as butterflies.

These control measures consist of increasing the value of μ_A and decreasing the value of α and β values. Thus, the parameters μ_A , α and β are no longer constants. We integrate the control actions through the impulsive differential equations as in work of Kolaye *et al.* [22]. Let $\mu_A^* = 1 - \mu_A$, we consider three parameters to determine the temporal evolution of $\varphi \in \{\mu_A^*, \alpha, \beta\}$, namely $\bar{\varphi}$ which represents the empirical value of φ and the growth rate r_φ .

Several types of (empirical) models can be chosen. For example, we choose the simplest ordinary differential equation:

$$\begin{cases} \frac{d\varphi}{dt} = r_\varphi(\bar{\varphi} - \varphi), \\ \varphi(0) = \varphi_0. \end{cases} \quad (4.4)$$

Assuming $r_\varphi > 0$, the analytical solution of (4.4) is given by

$$\varphi(t) = \bar{\varphi} - (\bar{\varphi} - \varphi_0)e^{-r_\varphi t}. \quad (4.5)$$

Thus, φ will converge quickly or not to the empirical value $\bar{\varphi}$ according to the value of r_φ . Assuming θ the proportion of *Busseola fusca* on which control methods are applied. We suppose that the efficiency of these control actions is related to θ so that the decrease in μ_A^* , α and β becomes more important as control methods are fully applied.

We assume that the control strategies are instantaneous (in time) and can be written as follows:

$$\Delta\varphi(t_i) = -\phi(\theta)\varphi(t_i),$$

where $\Delta\varphi(t_i) = \varphi(t_i^+) - \varphi(t_i)$ with $\varphi(t_i^+)$ designate the limit on the right φ at time t_i .

The control parameter $\phi \in [0, 1]$ is an augmented function with $\phi(0) = 0$ and $0 < \phi(1) < 1$, this implies that the control strategies are not perfectly efficient. The function ϕ can be defined as the following linear function

$\phi(x) = ax$ with $a \in]0, 1[$ or as a non-linear function, for example $\phi(x) = \frac{ax}{1+ax}$ with $a > 0$ where a is a parameter that can be related to the efficiency rate. Thus, assuming that the control actions start at day t_0 and occur every τ day, we obtain the following impulsive differential:

$$\begin{cases} \frac{d\varphi}{dt} = r_\varphi(\bar{\varphi} - \varphi), \\ \Delta\varphi(t_0 + n\tau) = -\phi(\theta)\varphi(t_0 + n\tau), n = 0, 1, 2, \dots \\ \varphi(0) = \varphi_0. \end{cases} \quad (4.6)$$

By the theory of impulsive differential equations [23], equation (4.6) is well defined and a single positive solution is unique. If θ is constant, *i.e.* $\theta(t) = \theta_0$, then, from simple calculations, we have the following result.

Proposition 4.1. *The periodic function:*

$$\varphi_{per} = \begin{cases} \bar{\varphi} - (\bar{\varphi} - \varphi)e^{-r_\varphi t}, & t \in [0, t_0] \\ \bar{\varphi} - (\bar{\varphi} - (1 - \phi(\theta))\varphi((n-1)\tau + t_0))e^{-r_\varphi(t - (n-1)\tau - t_0)}, & t \in [(n-1)\tau + t_0, n\tau + t_0], \\ & n = 1, 2, \dots \end{cases} \quad (4.7)$$

is the solution of equation (4.6).

Now, we will use each periodic solution $\mu_{A_{per}}^*$, α_{per} , and β_{per} of equation (4.7) in system (2.1). Replacing the constants parameters μ_A , α and β of system (4.7) by the functions $\mu_{A_{per}}(\mu_{A_{per}} = 1 - \mu_{A_{per}}^*)$, α_{per} and β_{per} respectively, one obtains the following system of non autonomous differential equations:

$$\begin{cases} \dot{E} = \alpha_{per}(t)(1 - \frac{E}{K})A - (\mu_E(T) + \gamma(T))E, \\ \dot{L} = \gamma(T)E - (\beta_{per}(t) + \mu_L(T) - (\epsilon\eta R + dL))L, \\ \dot{A} = \beta_{per}(t)L - \mu_{A_{per}}(t)A, \\ \dot{R} = \Lambda - \eta RL - \delta R. \end{cases} \quad (4.8)$$

In the sequel, $T = T(t)$ denotes the value of temperature at time t .

4.2. Well-posedness

System (4.8) must be consistent with the characteristics of a system of differential equations that represent a biologic population, in this case, the solution must be positive and bounded for all time.

System (4.8) can be written in the following form:

$$\dot{X} = f(X), \quad (4.9)$$

where

$$X = \begin{pmatrix} E \\ L \\ A \\ R \end{pmatrix} \quad \text{and} \quad f(X) = \begin{pmatrix} \alpha_{per}(t)(1 - \frac{E}{K})A - (\mu_E(T) + \gamma(T))E \\ \gamma(T)E - (\beta_{per}(t) + \mu_L(T) - \epsilon\eta R + dL)L \\ \beta_{per}(t)L - \mu_{A_{per}}(t)A \\ \Lambda - \eta RL - \delta R \end{pmatrix}.$$

The right hand side of system (4.9) is continuously and indefinitely differentiable on \mathbb{R}_+^4 . Then, by the Cauchy-Lipschitz theorem, system (4.9) provides a unique local positive solution with any initial conditions taken as $E(0) > 0$, $L(0) > 0$, $A(0) > 0$ and $R(0) > 0$.

We point out that the methods of analysis proposed in the case of the autonomous model of the first section are no longer appropriate given that our system is now non-autonomous. This is for example the case of the method of Van den Driessche and Watmough [18] for the computation of the threshold \mathcal{N}_0 .

Before performing analysis on the non-autonomous system, we must ensure the positivity and boundedness of solutions. We have the following result.

Proposition 4.2. *Let $X_0 = (E(0), L(0), A(0), R(0)) \in \mathbb{R}_+^4$ and $T_{\max} > 0$. Let us consider the maximum solution $X(t) = (E(t), L(t), A(t), R(t))$ of the Cauchy problem associated with system (3.13) on $[0, T_{\max}[$ of initial condition $(E(0), L(0), A(0), R(0))$, then for all $t \in [0, T_{\max}[$, $X(t) \in \mathbb{R}_+^4$.*

Proof. Let $(E(t), L(t), A(t), R(t)) \in \mathbb{R}_+^4$ be the maximum solution of the problem (4.8) associated with initial conditions $(E(0), L(0), A(0), R(0)) \in \mathbb{R}_+^4$.

Integrating each equations in system (3.13) from 0 to t yields

$$E(t) = \Sigma(t) \left(E(0) + \int_0^t \alpha_{per}(s) A(s) \Sigma(t) \right), \text{ where } \Sigma(t) = e^{-\int_0^t (\mu_E(T) + \gamma(T) + \frac{\alpha_{per}(s) A(s)}{K}) ds},$$

$$L(t) = \Delta(t) \left(L(0) + \int_0^t \gamma(T) E(s) \Delta(t) \right), \text{ where } \Delta(t) = e^{-\int_0^t (\beta_{per}(s) + \mu_L(T) - \epsilon \eta R(s) + dL(s)) ds},$$

$$A(t) = e^{-\int_0^t \mu_{A_{per}}(s) ds} \left(A(0) + \int_0^t \beta_{per}(s) L(s) e^{\int_0^s \mu_{A_{per}}(u) du} ds \right),$$

and

$$R(t) = e^{-\int_0^t (\delta + \eta L(u)) du} \left(R(0) + \int_0^t \Lambda e^{\int_0^s (\delta + \eta L(u)) du} ds \right).$$

Since the solution is continuous, there is a $\tilde{t} > 0$ such that $(E(t), L(t), A(t), R(t)) \in \mathbb{R}_+^4$ for all $t \in]0, \tilde{t}[$. Suppose by the absurd the existence of a finite τ defined by

$$\tau = \min \left\{ \tilde{t} + \zeta \in [0, T_{\max}[: \zeta > 0, (E(\tilde{t} + \zeta), L(\tilde{t} + \zeta), A(\tilde{t} + \zeta), R(\tilde{t} + \zeta)) \notin \mathbb{R}_+^4 \right\}.$$

According to equations $rE(t), L(t), A(t)$ and $R(t)$, one has $E(\tau) > 0, L(\tau) > 0, A(\tau) > 0$ and $R(\tau) > 0$ which contradicts the definition of τ . Hence, for all $t \in [0, T_{\max}[$, $(E(t), L(t), A(t), R(t)) \in \mathbb{R}_+^4$. This concludes the proof. \square

Definition 4.3. Consider a continuous and periodic function $f : \mathbb{R}_+ \rightarrow [0, 1]$ with period τ , we define

$$\bar{f} = \frac{1}{\tau} \int_0^\tau f(t) dt, \quad \bar{f} = \max_{0 \leq t \leq \tau} f(t) \text{ and } \underline{f} = \min_{0 \leq t \leq \tau} f(t).$$

As the consumption of the resource reduces the natural mortality of larvae, we set the positive parameter $\underline{s} = \underline{\mu}_L - \epsilon \eta \frac{\Lambda}{\delta}$ modeling the minimum natural mortality of larvae according to consumption of the resource.

Proposition 4.4. *The set*

$$\Gamma = \left\{ (E, L, A, R) \in \mathbb{R}_+^4 / E \leq K, \quad L \leq \frac{\bar{\gamma} K}{\underline{\beta}_{per} + \underline{s}}, \quad A \leq \frac{\overline{\beta}_{per} \bar{\gamma} K}{\underline{\mu}_{A_{per}} (\underline{\beta}_{per} + \underline{s})}, \quad R \leq \frac{\Lambda}{\delta} \right\} \quad (4.10)$$

is a positively invariant set for system (4.8).

Using the maximum and minimum values of time-dependent parameters of system (4.8), we can establish the proof of Proposition 4.4 similarly to the proof of Proposition 3.3.

4.3. Extinction of the pest

Here, we will establish the conditions to control and eradicate the pest in a maize field.

Theorem 4.5. *Let*

$$\mathcal{N}_{max} = \frac{\overline{\alpha_{per}} \overline{\beta_{per}} \overline{\gamma}}{\underline{\mu_{Aper}} (\underline{\mu_E} + \underline{\gamma}) (\underline{\beta_{per}} + \underline{s})}. \quad (4.11)$$

If $\mathcal{N}_{max} < 1$, then the spread of the pest in the field extinguishes with time. This means that $L(t) \rightarrow 0$ when $t \rightarrow \infty$.

Proof. Consider the inequality $R(t) \leq \frac{\Lambda}{8}$ verified for all $t > 0$. From the third equation of system (4.8), one has

$$\dot{A}(t) \leq \overline{\beta_{per}} \overline{L} - \underline{\mu_{Aper}} A, \quad (4.12)$$

where $\overline{L} = \limsup_{t \rightarrow +\infty} L(t)$. For all $t > 0$, taking $A(0) \leq \frac{\overline{\beta L}}{\underline{\mu_{Aper}}}$, the inequality (4.12) verifies

$$A(t) \leq \frac{\overline{\beta L}}{\underline{\mu_{Aper}}}, \quad \forall t > 0. \quad (4.13)$$

From the first equation, one has that

$$\dot{E}(t) \leq \overline{\alpha_{per}} \frac{\overline{\beta L}}{\underline{\mu_{Aper}}} - (\underline{\mu_E} + \underline{\gamma}) E, \quad (4.14)$$

which gives

$$E(t) \leq \frac{\overline{\alpha_{per}} \overline{\beta_{per}} \overline{L}}{\underline{\mu_{Aper}} (\underline{\mu_E} + \underline{\gamma})}, \quad \forall t > 0, \quad (4.15)$$

where $E(0) \leq \frac{\overline{\alpha_{per}} \overline{\beta_{per}} \overline{L}}{\underline{\mu_{Aper}} (\underline{\mu_E} + \underline{\gamma})}$.

Now, using equation (4.13) and (4.15), and from the second equation of system (3.13), one can deduce that

$$\dot{L}(t) \leq \frac{\overline{\gamma} \overline{\alpha_{per}} \overline{\beta_{per}} \overline{L}}{\underline{\mu_{Aper}} (\underline{\mu_E} + \underline{\gamma})} - (\underline{\beta_{per}} + \underline{s}) L. \quad (4.16)$$

Using the fluctuation method developed in references [24, 25], there exists a sequence t_n with $t_n \rightarrow \infty$ such that

$$\dot{L}(t_n) \rightarrow 0, \quad n \rightarrow \infty \quad \text{and} \quad L(t_n) \rightarrow \overline{L}, \quad n \rightarrow \infty.$$

Thus, one can deduce that

$$0 \leq \left(\frac{\overline{\gamma} \overline{\alpha_{per}} \overline{\beta_{per}}}{\underline{\mu_{Aper}} (\underline{\mu_E} + \underline{\gamma}) (\underline{\beta_{per}} + \underline{s})} - 1 \right) (\underline{\beta_{per}} + \underline{s}) \overline{L}.$$

With this in mind, one has that

$$(\mathcal{N}_{max} - 1) \left(\underline{\beta}_{per} + \underline{s} \right) \bar{L} \geq 0, \quad (4.17)$$

which implies that $\bar{L} = 0$ if $\mathcal{N}_{max} < 1$. This concludes the proof. \square

Thus, if $\mathcal{N}_{max} < 1$, then the pest is eradicated.

4.4. Persistence of the pest

Herein, we establish the conditions under which the pest permanently exists in a field.

Let

$$\mathcal{N}_{min} = \frac{\alpha_{per} \beta_{per} \gamma \tilde{\lambda}}{\mu_{A_{per}} \left(\underline{\mu}_E + \bar{\gamma} + \frac{\alpha_{per} \bar{A}}{K} \right) (\bar{\beta}_{per} + \bar{\mu}_L + d\bar{L})}, \quad (4.18)$$

where $\tilde{\lambda} = \frac{\underline{L}}{\bar{L}}$. We have the following result about the persistence of the pest within a maize field.

Theorem 4.6. *Assume that there exists $t_1 > 0$ such that $L(t_1) > 0$. If $\mathcal{N}_{min} > 1$, then the pest whose population dynamics is described by system (4.8) persists uniformly within the maize field.*

Proof. We will show by absurd assuming that there is $\xi > 0$ such that

$$\limsup_{t \rightarrow +\infty} L(t) \leq \xi.$$

From the the third equation of system (4.8), we have

$$\dot{A} \geq \underline{\beta}_{per} \underline{L} - \overline{\mu}_{A_{per}} A,$$

where $\underline{L} = \liminf_{t \rightarrow +\infty} L(t)$. Thus, taking $A(0) \geq \frac{\underline{\beta}_{per} \underline{L}}{\overline{\mu}_{A_{per}}}$, it comes that

$$A(t) \geq \frac{\underline{\beta}_{per} \underline{L}}{\overline{\mu}_{A_{per}}}, \quad \forall t \geq 0.$$

From the first equation of system (4.8), one has that

$$\dot{E}(t) \geq \underline{\alpha}_{per} \bar{A} - \left(\underline{\mu}_E + \bar{\gamma} + \frac{\alpha_{per} \bar{A}}{K} \right) E,$$

where $\bar{A} = \limsup_{t \rightarrow +\infty} A(t) = \frac{\overline{\beta}_{per} \bar{\gamma} K}{\mu_{A_{per}} (\bar{\beta}_{per} + \underline{s})}$. Thus, $\forall t \geq 0$,

$$E(t) \geq \frac{\underline{\alpha}_{per} \bar{A}}{\left(\underline{\mu}_E + \bar{\gamma} + \frac{\alpha_{per} \bar{A}}{K} \right)} \geq \frac{\alpha_{per} \beta_{per} \underline{L}}{\mu_{A_{per}} \left(\underline{\mu}_E + \bar{\gamma} + \frac{\alpha_{per} \bar{A}}{K} \right)}$$

Finally, from the second equation of system (4.8), one has that

$$\begin{aligned} \dot{L}(t) &\geq \underline{\gamma} \underline{E} - (\bar{\beta}_{per} + \bar{\mu}_L + d\bar{L}) L, \\ &\geq \frac{\underline{\gamma} \alpha_{per} \beta_{per} \underline{L}}{\mu_{A_{per}} \left(\underline{\mu}_E + \bar{\gamma} + \frac{\alpha_{per} \bar{A}}{K} \right)} - (\bar{\beta}_{per} + \bar{\mu}_L + d\bar{L}) L. \end{aligned} \quad (4.19)$$

With this in mind, one has that

$$\frac{dL(t)}{L(t)} \geq \frac{\gamma \alpha_{per} \beta_{per} \underline{L}}{\mu_{A_{per}} \left(\overline{\mu_E} + \overline{\gamma} + \frac{\overline{\alpha_{per} A}}{K} \right) \overline{L}} - (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L}).$$

Considering $t_1 \geq 0$, such that $L(t_1) > 0$, the resolution of equation (4.19) allows us to obtain the following inequality for all $t \geq t_1$:

$$\ln(L(t)) - \ln(L(t_1)) \geq \left(\frac{\gamma \alpha_{per} \beta_{per} \underline{L}}{\mu_{A_{per}} \left(\overline{\mu_E} + \overline{\gamma} + \frac{\overline{\alpha_{per} A}}{K} \right) \overline{L}} - (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L}) \right) (t - t_1),$$

which can be written as follows:

$$\ln \frac{L(t)}{L(t_1)} \geq \left(\frac{\gamma \alpha_{per} \beta_{per} \tilde{\lambda}}{\mu_{A_{per}} \left(\overline{\mu_E} + \overline{\gamma} + \frac{\overline{\alpha_{per} A}}{K} \right) (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L})} - 1 \right) (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L}) (t - t_1),$$

where $\tilde{\lambda} = \frac{\underline{L}}{\overline{L}}$. Thus

$$\frac{L(t)}{L(t_1)} \geq \exp \left((\mathcal{N}_{min} - 1) (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L}) (t - t_1) \right), \quad (4.20)$$

which implies that

$$L(t) \geq L(t_1) \exp \left((\mathcal{N}_{min} - 1) (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L}) (t - t_1) \right). \quad (4.21)$$

From equation (4.21), one has that

$$\lim_{t \rightarrow +\infty} L(t) \geq \lim_{t \rightarrow +\infty} L(t_1) e^{(\mathcal{N}_{min} - 1) (\overline{\beta_{per}} + \overline{\mu_L} + d\overline{L}) (t - t_1)}.$$

Then, it comes that

$$\limsup_{t \rightarrow +\infty} L(t) = +\infty, \quad (4.22)$$

which contradicts the fact that $L(t)$ is bounded. Then, one can conclude that if $\mathcal{N}_{min} > 1$, the *busseola fusca* pest is uniformly persistent in a maize field. This achieves the proof. \square

4.5. Numerical simulations

Here, we present the results of numerical simulations to illustrate and validate the theoretical results.

We first present the curves of the time evolution of *Busseola fusca* without taking into account temperature variation by using the same values for the non-temperature dependent parameters. After we plot the curves of the time evolution of *Busseola fusca* by inserting the temperature dependent parameters to illustrate the impact of temperature on the dynamics. To do so, we use the parameter values given in Table 5.

The actual data and the fitted temperature curve in Douala (Cameroon) for the period from 2005 to 2011 are shown in Figure 5.

TABLE 5. Parameter values used for numerical simulations.

Parameter	Value
$\underline{\alpha}$	10 number day ⁻¹
$\overline{\alpha}$	150 number day ⁻¹
$\underline{\gamma}$	0.2 day ⁻¹
$\overline{\gamma}$	0.6 day ⁻¹
$\underline{\beta}$	0.1 day ⁻¹
$\overline{\beta}$	0.7 day ⁻¹
K	1000 number
$\underline{\mu_E}$	0.01 day ⁻¹
$\overline{\mu_E}$	0.99 day ⁻¹
$\underline{\mu_L}$	0.5 day ⁻¹
$\overline{\mu_L}$	0.99 day ⁻¹
d	0.45 day ⁻¹ number ⁻¹
$\underline{\mu_A}$	0.25 day ⁻¹
$\overline{\mu_A}$	0.99 day ⁻¹
Λ	3 gr. day ⁻¹
δ	0.15 day ⁻¹
η	0.1 day ⁻¹
ϵ	0.05 day ⁻¹

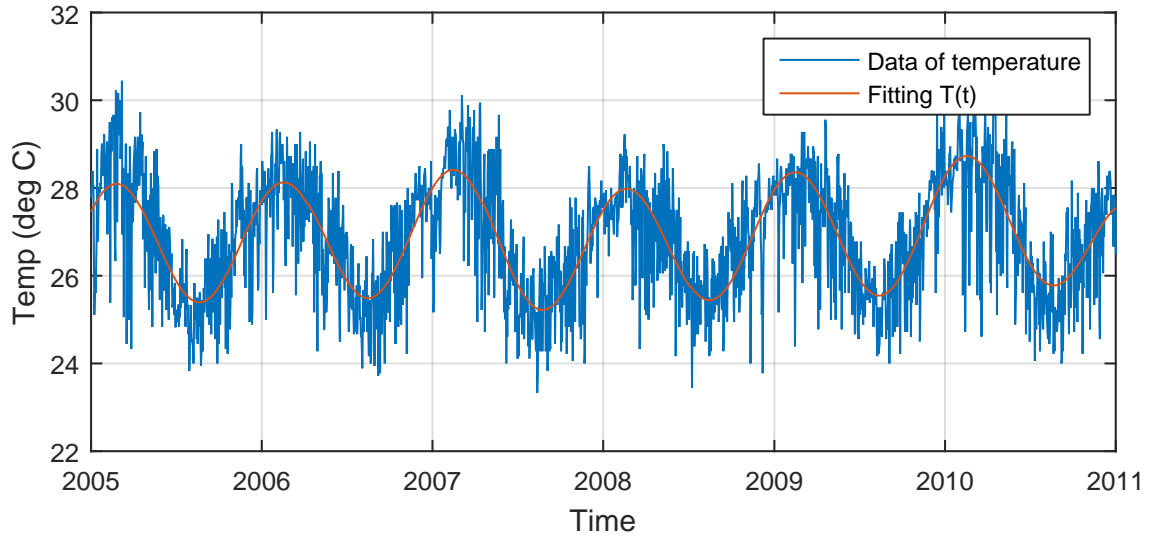


FIGURE 5. Temperature of Douala from 2005 at 2011 (Source: Postdam Institute of Climatology (PIK)).

We used the `cftool` in Matlab R2015a to fit the statistical data of temperature and we obtained

$$\begin{aligned}
 T(t) = & 82.27 \sin(0.0009097t + 0.7393) + 55.7 \sin(0.001133t + 3.683) \\
 & + 0.3052 \sin(0.003942t + 5.162) + 1.42 \sin(0.01734t + 0.7017) \\
 & + 0.1663 \sin(0.01001t + 0.4355).
 \end{aligned} \tag{4.23}$$

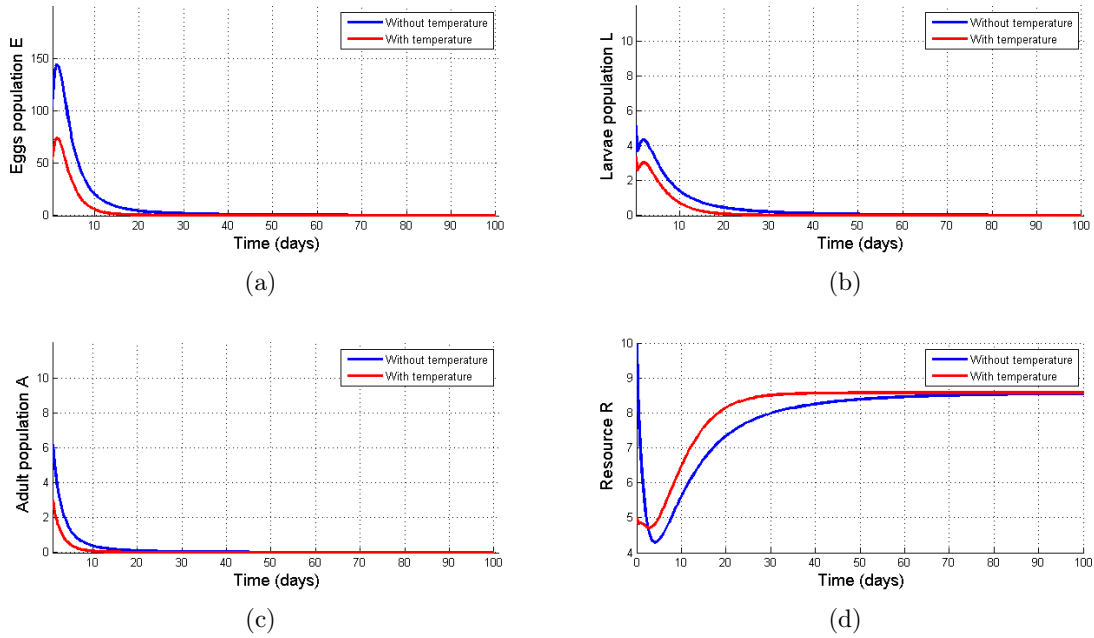


FIGURE 6. Simulation of system (2.1) without temperature (blue line) and with temperature (red line) when $\mathcal{N}_0 \leq \mathcal{N}_{max} < 1$ with the optimal temperature for hatching $T_{opt}^E = 23$ and for larval development $T_{opt}^L = 28$.

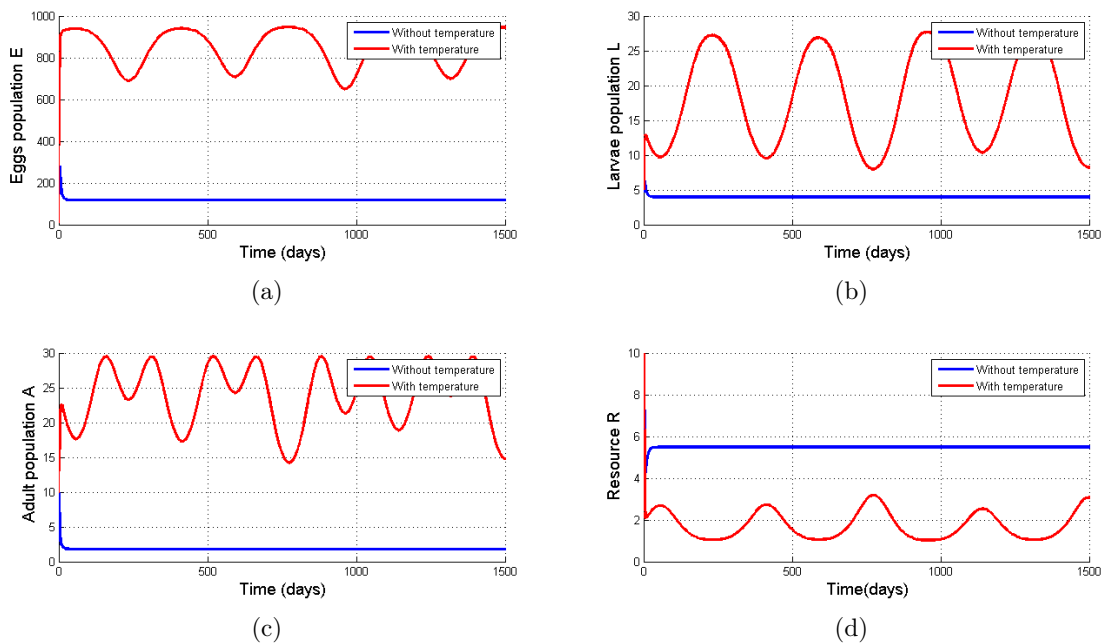


FIGURE 7. Simulation of system (2.1) without temperature (blue line) and with temperature (red line) when $\mathcal{N}_0 \geq \mathcal{N}_{min} > 1$ with the optimal temperature for hatching $T_{opt}^E = 23$ and for larval development $T_{opt}^L = 28$.

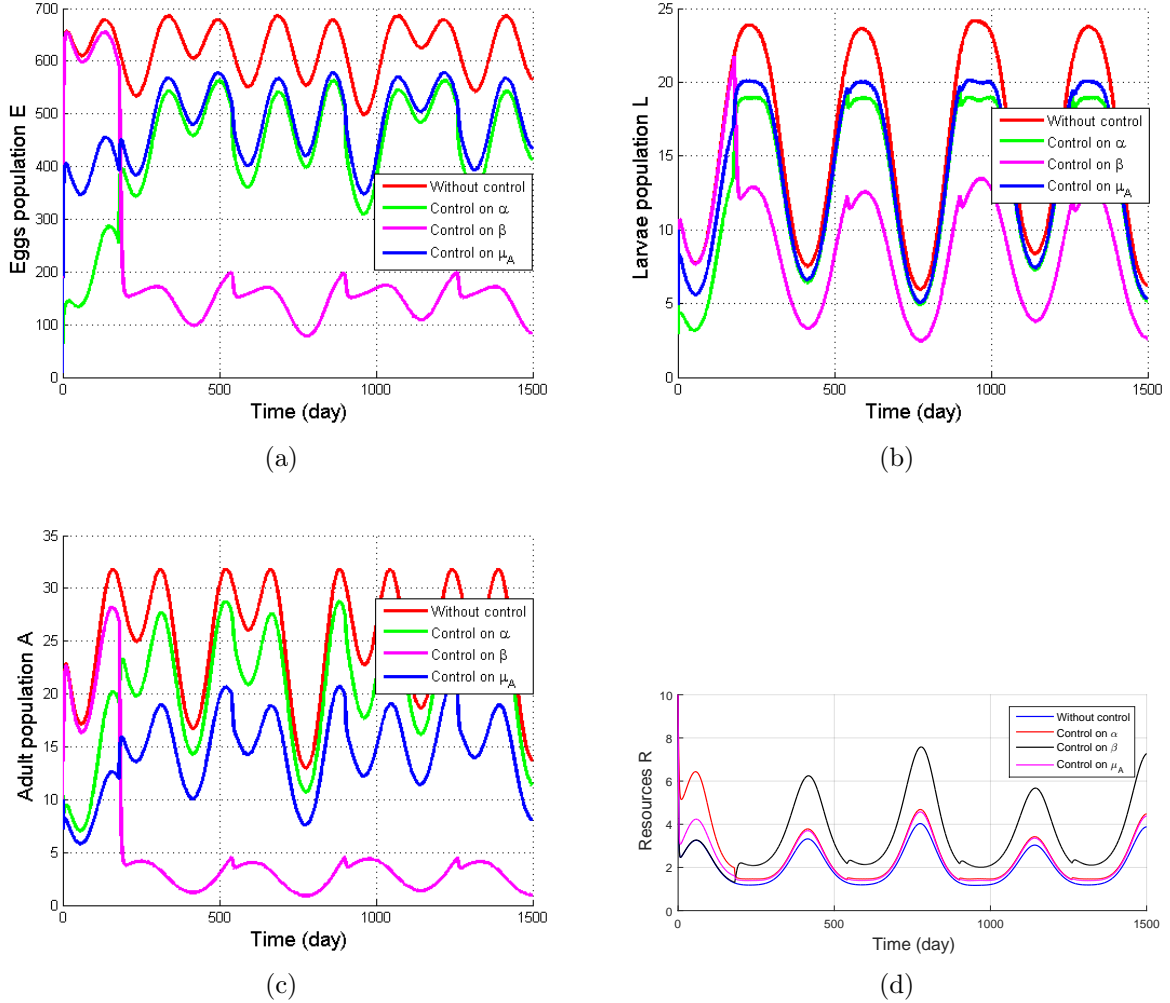


FIGURE 8. Variation in the number of individuals in *Busseola fusca* population by applying the three control strategies based on α , β and μ_A .

Figure 6 presents the asymptotic behavior of the population number of *B. fusca* without temperature variations (blue line) and with temperature variations (green line). The parameter values are modified in order to be in the case where $\mathcal{N}_0 \leq \mathcal{N}_{max} < 1$. So, we set $\underline{\mu}_A = 0.55$, $\underline{\mu}_E = 0.3$, $\underline{\gamma} = 0.03$, $\underline{\beta} = 0.03$, $\underline{\mu}_L = 0.53$, $\underline{\alpha} = 15$, $\underline{\gamma} = 0.089$, $\underline{\beta} = 0.02$ and $\underline{\alpha} = 30$ (so that $\mathcal{N}_{max} < 1$). All other parameter values are given in Table 5. It clearly appears that the pest disappears quickly in that condition.

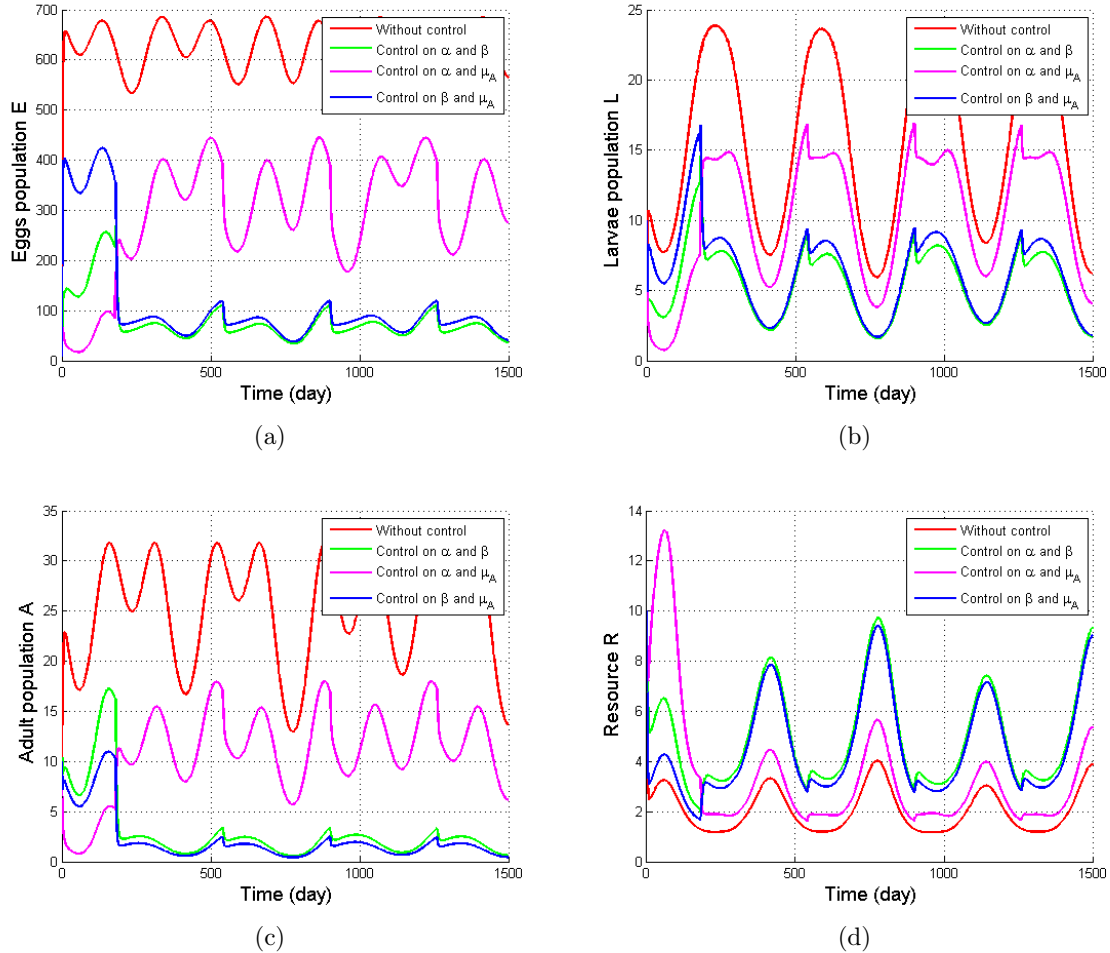
Figure 7 shows the asymptotic behaviors of the population dynamics of *Busseola fusca* without and with temperature variations when $\mathcal{N}_0 \geq \mathcal{N}_{min} > 1$. From this figure, it is observed that maize-pest persists in environment and that the temperature variation seems favouring the persistence of the pest.

We now run simulations to evaluate the effectiveness of different control strategies and the impact of temperature variation. We first consider the impact of each of three controlled parameters α , β and μ_A on the pest's population dynamics. We also consider the interaction of the three previous parameters on the evolution of the insect.

Figure 8 presents the asymptotic behaviour of the number of individuals of *Busseola fusca* by considering separately the impact of the different control strategies and the influence of temperature. It clearly appears

TABLE 6. Estimation of effect of each controls strategies on states of model (4.8).

	Eggs	Larvae	Adult	Resource
Without control	678	23	31	4.03
With contol on α	562 (17%)	19 (17%)	28 (10%)	4.68 (16%)
With control on β	199 (71%)	12 (48%)	4 (87%)	7.58 (88%)
With control on μ_A	577 (14%)	20 (13%)	20 (35%)	4.56 (13%)


 FIGURE 9. Variation in the number of population of *Busseola fusca* applying two control strategies at the same time.

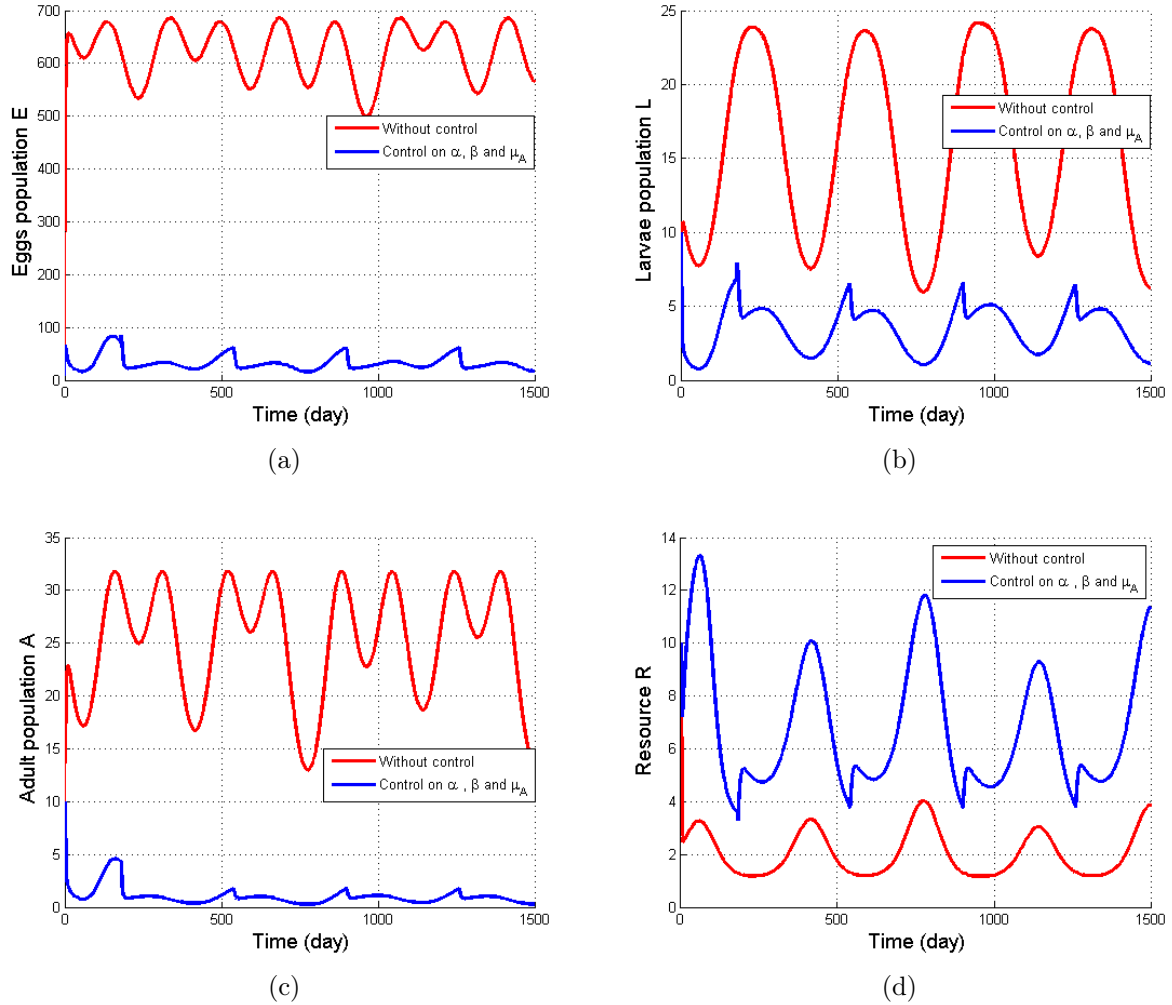
that the control action on the parameter β is much more efficient compared to other parameters. The percentage reduction of eggs, larvae and adults for each type of control action is given in the Table 6.

We now plot the curves to assess the overall effectiveness of various proposed control strategies. Table 7 summarizes the effectiveness of each pairwise combination of the three available control strategies, while Table 8 quantifies the impact of applying all three control strategies simultaneously.

Figure 10 shows the asymptotic behaviour of the number of individuals in the *Busseola fusca* population applying the three different control strategies at the same time. By analyzing the different curves, it can be

TABLE 7. Estimate of states due to double control.

	Eggs	Larvae	Adult	Resource
Without control	685	27	32	4
With control on α and β	112 (84%)	9 (67%)	3 (91%)	9.7 (143%)
With control on α and μ_A	444 (35%)	17 (37%)	18 (44%)	5.6 (41%)
With control on β and μ_A	120 (82%)	9 (67%)	2 (94%)	9.4 (135%)

FIGURE 10. Variation in the number of individuals in the *Busseola fusca* population with the impact of all proposed control strategies.

concluded that the application of the different strategies is much more effective in significantly reducing the number of *Busseola fusca* population individuals in a maize field.

Figure 11 shows the asymptotic behaviour of the number of individuals in the pest population when spraying discontinuously during the cropping period (*i.e.* spraying first the 1st month, 2nd month and 3rd month). Trajectories decrease similarly when control are applied in these three moments.

TABLE 8. Estimation of states under the action of general control.

	Eggs	Larvae	Adult	Resource
Without control	686	23	32	4.03
With control on α , β and μ_A	62 (91%)	6 (74%)	2 (94%)	11.8 (193%)

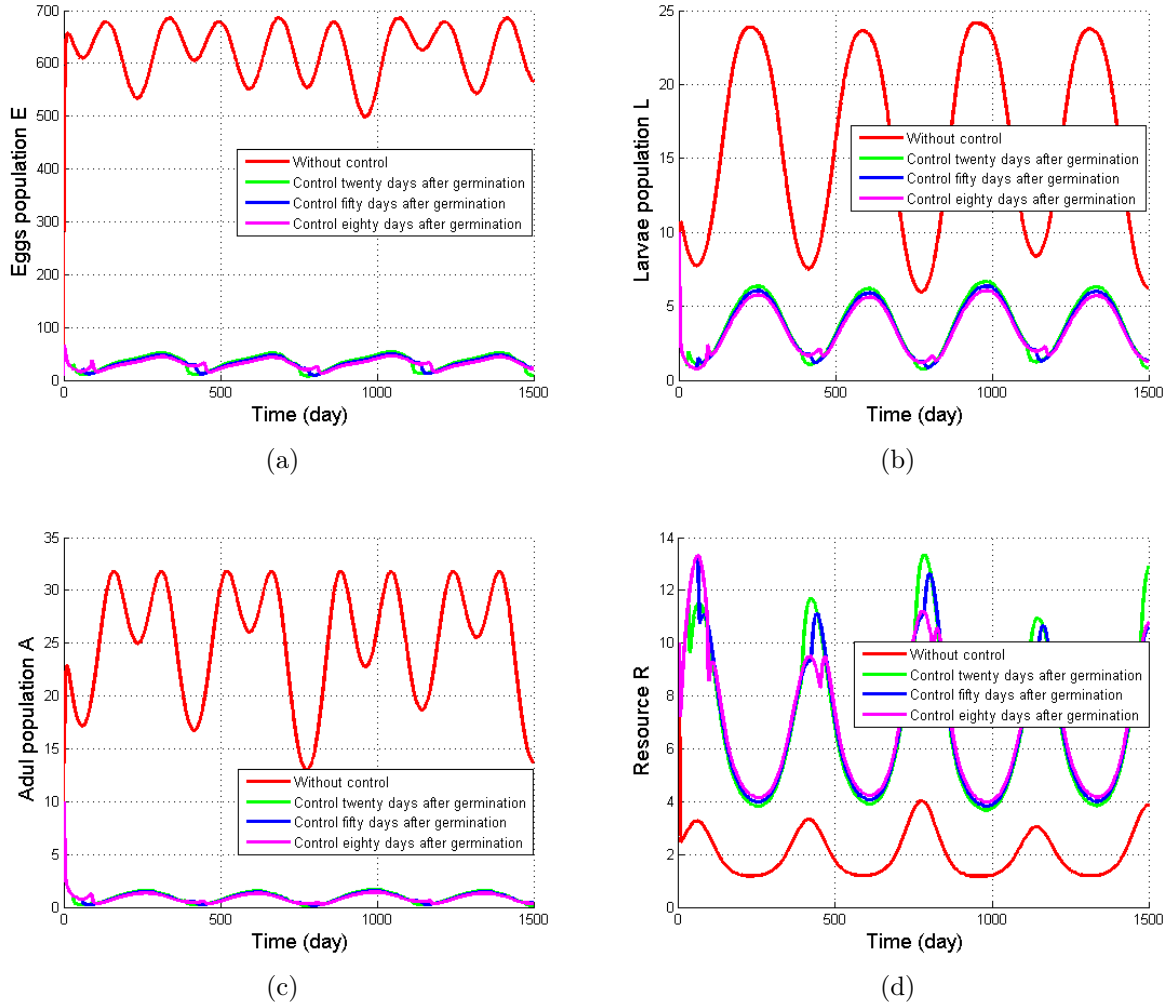


FIGURE 11. Variation in the number of individuals in the *Busseola fusca* population by discontinuously applying the proposed control strategies.

5. CONCLUSION

In this work, we proposed a mathematical model for the population dynamics of *B. fusca* taking into account climatic factors such as temperature variations. The objective was to assess the impact of temperature variations on the growth of a population of *B. fusca*: *Z. mays* pest. We first presented a mathematical model that can describe the temporal evolution of the population dynamics of *B. fusca* based on his life cycle. A qualitative analysis of the model has been presented. We have shown that the problem is well posed. We derived the basic

offspring number \mathcal{N}_0 , and shown that when $\mathcal{N}_0 < 1$ the trivial equilibrium is globally asymptotically stable, while when $\mathcal{N}_0 > 1$ the trivial equilibrium point is unstable and there exists a unique non trivial equilibrium point which is globally asymptotically stable in the positive orthant minus the trivial equilibrium point. Numerical simulations have been presented to illustrate and support analytical results. The sensitivity analysis of the model has been also investigated. We found that the parameter representing the eggs hatching rate influence more state variables of the system. After, we extended the proposed model by taking into account the the impact of temperature variations on the growth of population of *B. fusca*. Three control strategies are incorporated in the model of the population of *B. fusca*.

We provided also a thorough analytic study of model obtained. Theoretically, we find two explicit thresholds parameters \mathcal{N}_{max} and \mathcal{N}_{min} that bound the basic offspring number \mathcal{N}_0 such that $\mathcal{N}_{max} \leq \mathcal{N}_0 \leq \mathcal{N}_{min}$, and use them to prove the extinction and the persistence of the pest within a *Z. mays* field. We prove analytically and by numerical simulations that *B. fusca* persists uniformly within a *Z. mays* field when $\mathcal{N}_{min} > 1$ and tends to disappears within a *Z. mays* field when $\mathcal{N}_{max} < 1$.

By analyzing the asymptotic behavior of the pest population *B.fusca*, we found that the temperature of the environment has a significant impact on the evolution of the pest. Temperature significantly affects the development and reproduction of *B. fusca*. The effect of the increase and decrease in temperature has an impact on the pest and therefore on the production of *Z.mays*. The strategy used by the insect *B. fusca* to survive in periods when the temperature is not favorable is to enter diapause, a period during which no activity occurs. It can therefore be assumed that if temperatures increase or decrease due to climate change that does not adversely affect corn growth, there will be less damage to corn crops.

We also applied control on certain parameters. For each parameter considered, we analyzed the impact of the control on the asymptotic behavior of the pest population. We found that spraying insecticides to kill larvae and destroying residues after harvest significantly reduced the number of individuals in the pest population *B. fusca*. We also found that the joint application of different control strategies considered has a significant impact as it can significantly reduce the pest population in a maize field. Control applied at the larval stage is more effective but care should be taken not to apply the control for larvae at advanced growth stages. As the larvae at advanced stages are well protected inside the stem this will lead to the inefficiency of certain methods used to slow down the evolution of the insect. As the eggs and larvae of *B. fusca* are visible on the maize plants, this will make it possible to observe any attacks on the crop and to effectively apply control methods.

DATA AVAILABILITY STATEMENT

No new data/codes were created or analyzed in this study.

REFERENCES

- [1] D. Byerlee and P.W. Heisey, Past and potential impacts of maize research in sub-Saharan Africa: a critical assessment. *Food Policy* **3** (1996) 255–277.
- [2] A. Polaszek and G. Delvare, Les Foreurs des tiges de céréales en Afrique : Importance économique, systématique, ennemis naturels et méthodes de lutte. [Cereal stem borers in Africa: economic importance, systematics, natural enemies and methods of control], CIRAD pour la version française (2000).
- [3] N.A. Bosque-Pérez and F. Schulthess, Maize: West and Central Africa, In African Cereal Stem Borers: Economic Importance, Taxonomy, Natural Enemies and Control, edited by A. Polaszek. CTA/CABI International, Wallingford (1998).
- [4] M. Sezonlin, *Phyogéographie et génétique des populations du foreur de tiges de céréales Busseola fusca (Fuller) (Lepidoptera, Noctuidae) en Afrique Sub-Saharienne, implications pour la lutte biologique contre cet insecte*. [Phyogeography and genetics of population of the cereal stem borer *Busseola fusca* (Fuller) (Lepidoptera, Noctuidae) in sub-Saharan Africa, implications for the biological control of this insect]. Thèse de doctorat de l'Université de Paris XI-Orsay (2006).
- [5] K.M. Harris and K.E. Nwanza, *Busseola fusca* (Fuller), the African Maize Stalk Borer: A Handbook of Information, Information Bulletin No. 33. ICRISAT (1992).

- [6] J. Glatz, The effect of temperature on the development and reproduction of *Busseola fusca* (Lepidoptera : Noctuidae), dissertation submitted in fulfilment of the requirements for the degree Magister Scientiae in Environmental Sciences at the Potchefstroom Campus of the North-West University (2015).
- [7] E.J. Usua, Temperature and relative humidity effects on the development of immature stages of the maize stem borers *Busseola fusca* and *Sesamia calamistis*. *J. Econ. Entomol.* **61** (1968) 1091–1093.
- [8] A. Nimalan and N.I. Arran, Mathematical modelling for the control of infectious diseases. Short courses, 6–15 September 2021. Department of Infectious Disease Epidemiology, Imperial College London, London.
- [9] K.M. Harris and K.E. Nwanze, *Busseola fusca* (Fuller), the African Maize Stalk Borer: A Handbook of Information. International Crops Research Institute for the Semi-Arid Tropics (1992).
- [10] K.M. Harris and K.E. Nwanze. *Busseola fusca* (Fuller), the African Maize Stalk Borer: A Handbook of Information. International Crops Research Institute for the Semi-Arid Tropics (1992).
- [11] M. Muma, S.J. Gumiere and A.N. Rousseau, Analyse de sensibilité globale du modèle cathy aux propriétés hydrodynamiques du sol d'un micro-bassin agricole drainé. [Global sensitivity analysis of the cathy model with hydrodynamic properties of the soil of a drained agricultural micro-basin]. *Hydrol. Sci. J.* **59** (2014) 1606–1623.
- [12] A. Saltelli, *et al.*, A quantitative model-independent method for global sensitivity analysis of model output, technometrics. *J. Statist. Phys. Chem. Eng. Sci.* **41** (1999) 39–56.
- [13] A. Saltelli, *et al.*, Sensitivity Analysis in Practice. A Guide to Assessing Scientific Models. John Wiley & Sons Ltd, New York (2004).
- [14] C.W. Mally, The maize stalk borer, *Busseola fusca* (Fuller). *Bull. Depart. Agric. Union South Afr.* **3** (1920) 111.
- [15] J.A. Jacquez and C.P. Simon, Qualitative theory of compartmental systems. *Siam Rev.* **35** (1993) 43–79.
- [16] A. Berman and R.J. Plemmons, Nonnegative Matrices in the Mathematical Sciences. Society for Industrial and Applied Mathematics (1994).
- [17] L. Mailleret, *Stabilisation Globale de Systèmes Dynamiques Positifs Mal Connus. Applications en Biologie*. [Global Stabilization of unKnown Positive Dynamical Systems. Applications in Biology.] PhD thesis, Université Nice Sophia Antipolis (2004).
- [18] P. Van den Driessche and J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Math. Biosci.* **180** (2002) 29–48.
- [19] O. Diekmann, J.A.P. Heesterbeek and M.G. Roberts, The construction of next generation matrices for compartmental epidemic models. *J. R. Soc. Interface* **7** (2009) 873–885.
- [20] J Carr, Applications Centre Manifold Theory. Springer-Verlag (1981).
- [21] C. Castillo-Chavez and B. Song, Dynamical models of tuberculosis and their applications. *Math. Biosci. Eng.* **1** (2004) 361–404.
- [22] G. Kolaye, I. Damakoa, S. Bowong, R. Houe and D. Békollé, A mathematical model of cholera in a periodic environment with control actions. *Int. J. Biomath.* **13** (2008) 2050025.
- [23] D. Bainov and P. Simeonov, Impulsive Differential Equations: Periodic Solutions and Applications, Vol. 66. CRC Press (1993).
- [24] H.I. Freedman and P. Moson, Persistence definition and their connections. *Proc. Am. Math. Soc.* **109** (1990) 1025–1033.
- [25] H.L. Freedman, S. Ruan and M. Tang, Uniform persistence and flows near a closed positively invariant set. *J. Dyn. Differ. Equ.* **6** (1994) 583–600.



Please help to maintain this journal in open access!

This journal is currently published in open access under the Subscribe to Open model (S2O). We are thankful to our subscribers and supporters for making it possible to publish this journal in open access in the current year, free of charge for authors and readers.

Check with your library that it subscribes to the journal, or consider making a personal donation to the S2O programme by contacting subscribers@edpsciences.org.

More information, including a list of supporters and financial transparency reports, is available at <https://edpsciences.org/en/subscribe-to-open-s2o>.

APPENDIX A. CASTILLO-CHAVEZ AND SONG THEOREM [21]

Theorem A.1. Consider the following general system of ordinary differential equations with a parameter Φ :

$$\frac{dx}{dt} = f(x, \Phi), \quad f : \mathbb{R}^n \times \mathbb{R} \text{ and } f \in C^2(\mathbb{R}^n, \mathbb{R}); \quad (\text{A.1})$$

where 0 is an equilibrium point of the system (that is, $f(0, \Phi) \equiv 0$ for all Φ) and assume

A1: $A = D_x f(0, 0) = \left(\frac{\partial f_i}{\partial x_j}(0, 0) \right)$ is the linearization matrix of system (A.1) around the equilibrium 0 with Φ evaluated at 0 . Zero is a simple eigenvalue of A and other eigenvalues of A have negative real parts;

A2: Matrix A has a right eigenvector v and a left eigenvector w (each corresponding to the zero eigenvalue). Let f_k be the k^{th} component of f and

$$a = \sum_{k,i,j=1}^n v_k w_i w_j \frac{\partial^2 f_k}{\partial x_i \partial x_j}(0, 0) \text{ and } b = \sum_{k,i=1}^n v_k w_i \frac{\partial^2 f_k}{\partial x_i \partial \Phi}(0, 0),$$

then, the local dynamics of the system around the equilibrium point 0 is totally determined by the signs of a and b .

1. $a > 0, b > 0$. When $\Phi < 0$ with $|\Phi| \ll 1$, 0 is locally asymptotically stable and there exists a positive unstable equilibrium, when $0 < \Phi \ll 1$, 0 is unstable and there exists a negative, locally asymptotically stable equilibrium;
2. $a < 0, b < 0$. When $\Phi < 0$ with $|\Phi| \ll 1$, 0 is unstable, when $0 < \Phi \ll 1$, 0 is locally asymptotically stable equilibrium, and there exists a positive unstable equilibrium;
3. $a > 0, b < 0$. When $\Phi < 0$ with $|\Phi| \ll 1$, 0 is unstable and there exists a locally asymptotically stable negative equilibrium, when $0 < \Phi \ll 1$, 0 is stable, and a positive unstable equilibrium appears;
4. $a < 0, b > 0$. When Φ changes from negative to positive, 0 changes its stability from stable to unstable. Correspondingly a negative unstable equilibrium becomes positive and locally asymptotically stable.