GAS EFFECT FOR OBLIQUE AND CONICAL SHOCK WAVES AT HIGH TEMPERATURE

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Abstract. The work focuses to develop a new numerical calculation program for determining the gas effect at high temperature instead air on the calculation of the oblique and conical shock waves parameters and make applications for various external and internal aerodynamics problems like, the calculation of the suitable intake adaptation parameters, dihedron and cone wave drag, aerodynamic coefficients of a pointed supersonic airfoil and oblique shock reflection without forgetting others no less important like the detonation propulsion and the dust explosion applications, where the high temperature gas effect is very important. All this for future aerodynamics (gas dynamics) like the phenomenon of climate change in the near and far future because of the enlargement progressive of the layer ozone hole which will lead to an increase in the temperature of the ambient medium, and by the environment pollution by the shining of the waste which will cause a new decomposition of gases from the ambient environment. Another interesting application for actual aerodynamics (gas dynamics) is the performance of tests in wind tunnels supplied by a combustion chamber making a reaction of gases giving a gas with new thermodynamics parameters which is not necessarily air. To make a calculation, the selected gases are H_2 , O_2 , N_2 , CO , CO_2 , H_2O , NH_3 , CH_4 and air. All shock parameters depend on the stagnation temperature, upstream Mach number, the thermodynamics of the used gas, dihedron and cone deviation and others parameters. The specific heat at constant pressure varies with the temperature and the selected gas. Gas is still considered as perfect. It is calorically imperfect, and thermally perfect, less than the molecules dissociation threshold. A comparison between the parameters of each gas and air is presented to choose the suitable gas witch giving good performances as required by design parameters instead air.

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1. INTRODUCTION

Gases play an important role in the aerodynamics (gas dynamics) for aeronautical and aerospace's vehicles. They influence the behavior of the supersonic flows and in particular on all thermodynamics and physical

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parameters. The *NSW*, *OSW* and *CSW* are a physical phenomenon appeared in nature to only supersonic flows when conditions are applied [2, 3, 13, 25].

In a supersonic nozzle when $P_E < P_a$, a *NSW* will develop in interior of the nozzle. The position and the parameters through the shock depend on P_E/P_a . In this case, the flow is subsonic, and it will affect the C_F of the nozzle [22, 25].

The attached *OSW* and *CSW* will respectively appear around the two-dimensional and axisymmetric pointed obstacles in a supersonic flow [18, 19, 27]. They have a very small thickness, so that they can be considered as mathematical discontinuity [2, 3]. Therefore, all physical parameters have a discontinuity through the shock except the total temperature.

The work done by [1–3, 13, 18, 19, 22, 24, 25, 27] for the *NSW*, *OSW* and *CSW* parameters are made by a calorically perfect ideal gas, *i.e.* the C_P is constant, and does not depend on the temperature. It gives good results for low T_0 , M_1 and φ , which does not, exceed 240 K, 2.00 and 10° respectively. This model for gas is called by *PG* model.

Air is frequently used as a gas in external and internal propulsion flows for all aeronautical, aerospace or even wind tunnel applications, especially for shock waves calculation, as it exists in quantity in nature [1–3, 12, 13, 16–19, 22, 24, 25, 27, 31].

In [5, 7, 8], the authors have developed the first study on the application of the propulsion gas effect on the conception of supersonic nozzle *MLN* 2D [5], *MLN* axisymmetric [7] and *PN* 2D [8] to *HT*. For the studies in [5, 7, 8] to be carried out, reference [9] always deals with the gas effect on the Prandtl–Meyer function at *HT*.

A study on the propellant gas effect at *HT* on the parameters through a *NSW* in non-adaptation regime of a supersonic nozzle is the goal of [9].

Others important applications for oblique shock wave are extended to present some problems for oblique detonation process. Modeling aluminum particle-air detonation is very difficult because the combustion phenomenon is shock-induced, and there are multi-phase heat release and transfer in supersonic flows. The reference [28] presents 2D cellular Aluminum particle-air detonations are simulated with the realistic heat capacity model and its effects on the detonation featured parameters, such as the detonation velocity and cell width, are analyzed.

The reference [30] provides a numerical investigation of the oblique detonation induced by a finite wedge in hydrogen-air mixtures. The Navier Stocks equations with a chemical model are solved to simulate the flow field characteristic of an oblique detonation wave. The paper summarizes the effects some parameters like the equivalence ratios, expansion waves, the initiation characteristics, wave structures, density, and temperature distributions on the oblique detonation wave.

The reference [29] presents two structures with smooth and abrupt transitions simulated by the two-dimensional Euler equations and a two-step induction–reaction model, and their interactions with the cowl-induced expansion wave are investigated.

The aim of this work is to develop a new computational program to study the effect of thermodynamics gas on the *OSW* and *CSW* parameters at *HT* to make a suitable choice of gas which gives good performances toward the shock for aeronautical and aerospace construction, and to solve the future aerodynamics (gas dynamics) problems on the increase of the ambient temperature caused by the increase in holes of the ozone layer, and by the radical and progressive change of the chemical composition of gas of the ambient environment presented by the burning of waste whose volume increases considerably every day, and consequently make applications for various external and internal aerodynamics problems like, the calculation of the suitable intake adaptation parameters, dihedron and cone wave drag, aerodynamic coefficients of a pointed supersonic airfoil and oblique shock reflection. Other interested problem can be solved by the use of this work like the oblique detonation process.

Among the advantages of our elaborate program is that a remarkable improvement on the method of solving conservation equations by the method of Runge Kutta of order 10 which is an important method for the problems for initial condition will be indicated in the paper. While in references [12, 18, 19] use the finite difference method for the obtained differential equations, which is a rather heavy and slightly precise method.

TABLE 1. Coefficients of $C_P(T)$ and R for some gases.

N	Gas	a' J/(K mol)	b' J/(K ² mol)	c' JK/mol	R J/(kg K)
1	H ₂	27.28	3.26	0.50	4157.25
2	O ₂	29.96	4.18	-1.67	259.828
3	N ₂	28.58	3.76	-0.50	296.946
4	CO	28.41	4.10	-0.46	296.946
5	CO ₂	44.22	8.79	-8.62	188.965
6	H ₂ O	30.54	10.29	0.08	461.916
7	NH ₃	29.75	25.10	-1.55	489.088
8	CH ₄	23.64	47.86	-1.92	519.656
9	Air	Polynomial of 9th degree [5, 7-9, 12, 31]			287.102

FIGURE 1. OSW illustration.

At high T_0 , M_1 and φ applications which exceed respectively 240 K, 2.00 and 10° , the PG model do not gives good results and a correction is necessary to this model by developing a new and adequate model called by HT model for this application, which answers the behavior of the flow in this case. The problem occurs in the energy equation, where $C_P(T)$ depends on the temperature, lower than threshold dissociation of the molecules [12, 16, 17, 31] which is not the case for the PG model, where C_P is considered as constant [1-3, 22, 24, 25]. The shock is characterized by the determination of the thermodynamic parameters and the Mach number through the shock as well as the increase of the entropy.

Among several gases existed in nature, The H₂, O₂, N₂, CO, CO₂, H₂O, NH₃, CH₄ and air are chosen in this study [4-6, 8-10, 14, 20, 21]. The gas is considered as perfect. Including the state equation ($P = \rho RT$) is still valid. It is considered calorically imperfect seen the change in $C_P(T)$ varies with temperature and thermally perfect because the dissociation of molecules is not reached. For the chosen gas is found in a variation according to the relation (1.1) of $C_P(T)$. The coefficients of this function and the constant R of the gas are shown in Table 1 [4, 6, 8-10, 14, 20, 21, 23].

$$C_P(T) = a' + b'T + \frac{c'}{T^2}. \quad (1.1)$$

Concerning the air, the variation of $C_P(T)$ is chosen as a polynomial of 9th degree [4, 5, 7-9, 12].

The function $C_P(T)$ is found in Joule/(mol K). One needs to convert it to the Joule/(kg K). For this we used the relationship (1.2).

$$C_P \left(\frac{\text{Joule}}{\text{kg K}} \right) = C_P \left(\frac{\text{Joule}}{\text{mol K}} \right) \times \frac{R}{8.3145}, \quad (1.2)$$

2. OSW FORMULATION AT HT

Figure 1 represents a general diagram of an OSW on a dihedron inclined by an angle ψ and the envisaged parameters.

The determination of the parameters (M_2 , β , T_2/T_1 , ρ_2/ρ_1 and P_2/P_1) by (2.1), (2.2), (2.3) and (2.4) through the *OSW* [16, 17], in the first step, The *HT* model presented in [12, 16, 17, 27] has been improved after making a correction to the relation between β , ψ and M_1 , since the authors in these references have used the equation designed for the *PG* model, at constant C_P [1, 3], given the difficulty of finding an analytic form, which gives results far from reality, and which does not respond to the behavior of *HT* flow. The said equation is the most interesting in the calculation of the *OSW* parameters, since all the other parameters depend on β , ψ and M_1 .

The relationships are summarized as follows [16, 17, 27]:

$$M_2^2 = \frac{1 - \frac{P_2}{P_1}}{\sin^2(\beta - \psi) \times \gamma(T_2) \times \left[1 - \frac{\rho_2}{\rho_1}\right] \frac{P_2}{P_1}}, \quad (2.1)$$

$$\frac{T_2}{T_1} = \frac{\frac{\rho_2}{\rho_1} - M_1^2 \sin^2(\beta) \times \gamma(T_1) \times \left(1 - \frac{\rho_2}{\rho_1}\right)}{\left(\frac{\rho_2}{\rho_1}\right)^2}, \quad (2.2)$$

$$\frac{\rho_2}{\rho_1} = \frac{\tan(\beta)}{\tan(\beta - \psi)}, \quad (2.3)$$

$$\frac{P_2}{P_1} = \left[\frac{\rho_2}{\rho_1}\right] \left[\frac{T_2}{T_1}\right]. \quad (2.4)$$

Two solutions can be found depending on the value of the obtained M_2 , which implies that all the physical parameters will admit two solutions. If $M_2 \geq 1$, we get a weak shock. If $M_2 < 1$, we will have a strong shock. In general, the weak shock that occurs in nature [2, 3, 22, 25].

Before the determination of the parameters through the *OSW*, it is necessary to determine, in the first place, the angle β corresponding to ψ and M_1 and the thermodynamics of the used gas. In this work, we will determine the β deviation with high accuracy according to the real *HT* model by applying the principle of *HT NSW* [31].

By varying M_1 , T_0 , ψ and the *used gas*, one can determine β , M_2 , T_2/T_1 , P_2/P_1 and ρ_2/ρ_1 through an *OSW*.

3. *CSW* FORMULATION AT *HT*

Figure 2 represents a general diagram of a *CSW* and the physical parameters presented as well as the adaptation of the cylindrical coordinates.

First, the parameters must be determined just after the shock by the numerical resolution of the equations (2.1), (2.2), (2.3) of *OSW* [12]. The second step consists in finding the parameters of the isentropic flow in the region just after the shock and the cone surface corresponding to θC , by numerical resolution of the differential equations (3.1), (3.2) and (3.3) coupled to three variables (V_r , V_θ and T) simultaneously by the adaptation of the Runge Kutta method of order 10 [11, 15, 23] with initial condition. As a result, all other parameters M_C , T_C/T_0 , ρ_C/ρ_{02} and P_C/P_{02} on the cone surface will be determined. In this case, the isentropic parameters between the shock and the cone surface depend on the angle θ .

$$\frac{dV_r}{d\theta} = V_\theta, \quad (3.1)$$

FIGURE 2. *CSW* illustration.

$$\frac{dV_\theta}{d\theta} = \frac{a(T) [2 V_r \tan(\theta) + V_\theta] - V_\theta^2 V_r \tan(\theta)}{\tan(\theta) [V_\theta^2 - a^2(T)]}, \quad (3.2)$$

$$\frac{dT}{d\theta} = \frac{V_\theta a(T) [V_r \tan(\theta) + V_\theta]}{\tan(\theta) C_P(T) [a^2(T) - V_\theta^2]}, \quad (3.3)$$

$$V^2 = V_r^2 + V_\theta^2, \quad (3.4)$$

$$V = \frac{M}{a}, \quad (3.5)$$

where

$$a(T) = \sqrt{\gamma(T) R T}, \quad (3.6)$$

$$\gamma(T) = \frac{C_P(T)}{C_P(T) - R}. \quad (3.7)$$

By varying M_1 , T_0 , θ_C and the *gas used*, we can find all the parameters β , ψ , M_2 , T_2/T_1 , ρ_2/ρ_1 and P_2/P_1 through a *CSW*.

4. SHOCK INTENSITY AND ENTROPY JUMP

The total pressures ratio and the entropy jump through the *OSW* or *CSW* can be determined respectively by the following relation [2, 3]:

$$\frac{P_{02}}{P_{01}} = \left[\frac{P_2}{P_1} \right] \left[\frac{P_1}{P_{01}} \right] / \left[\frac{P_2}{P_{02}} \right], \quad (4.1)$$

$$\frac{\Delta S_{12}}{R} = -\text{Log} \left(\frac{P_{02}}{P_{01}} \right). \quad (4.2)$$

We notice that $T_{02} = T_{01} = T_0$, $\rho_{02} \equiv \rho_{01}$ and $P_{02} \equiv P_{01}$.

FIGURE 3. Distance for an adapted intake.

5. ISENTROPIC PARAMETERS

The isentropic values [32] can therefore be determined immediately after the shock and on the cone surface by the relations (5.1), (5.2) and (5.3) respectively when $(M = M_2, T = T_2, \rho_0 = \rho_{02}, P_0 = P_{02})$ or $(M = M_C, T = T_C, \rho_0 = \rho_{02}, P_0 = P_{02})$.

$$M(T) = \frac{\sqrt{2H(T)}}{a(T)}, \quad (5.1)$$

$$\frac{\rho}{\rho_0} = \exp\left(-\int_T^{T_0} \frac{CP(T)}{a^2(T)} dT\right), \quad (5.2)$$

$$\frac{P}{P_0} = \left[\frac{T}{T_0}\right] \left[\frac{\rho}{\rho_0}\right], \quad (5.3)$$

With [23]

$$H(T) = \int_T^{T_0} C_P(T) dT. \quad (5.4)$$

6. APPLICATIONS

The flow out of an adaptation in a supersonic nozzle is a good example for *OSW* application. When $P_E < P_a$, *OSW* will be to occur in the interior of the nozzle, and *ME* will no longer supersonic, and necessarily affects the nozzle C_F .

The determination of λ of a supersonic aircraft intake engine is a good example for *OSW* and *CSW* applications.

Figure 3 shows a diagram of a suitable air intake. The initial position of the dihedral or central cone is given when its leading edge is on the same vertical line as the retraction of the nacelle.

In order for the air intake to be adapted, for not having the reflection of the shock wave inside the engine, the distance of the displacement of the dihedral or the central cone must be calculated by the following relation:

$$\lambda = \frac{d}{h} = \frac{1}{\tan(\beta)}. \quad (6.1)$$

The value of β depends on M_1, T_0, φ , the *gas used* and the dihedron or cone application.

If the distance λ given by (6.1) is not respected, the shock wave propagates inside the motor and consequently there will be a considerable increase in the drag force, which gives the adaptation of the intake is needed to decrease the drag and to have favorite's parameters after the shock.

FIGURE 4. Shock wave reflection in an unsuitable air intake.

FIGURE 5. Supersonic pointed curved non symmetrical airfoil.

In the case where the air intake is not suitable, we will therefore have the configuration shown in Figure 4 according to the lip shape of the nacelle with a weak *OSW* reflection. The parameters in the zone 3 can be calculated in the same way as the calculation in the zone 2 after the determination of β_3 and M_3 in the zone 3. The shock wave will be reflected by an angle σ with respect to the horizontal wall as shown in Figure 4. However $\sigma = \beta_3 - \psi$. Continuous reflection will be observed up to the motor exit. Then to avoid the reflection of the shock wave, the central body of the air intake must be placed according to Figure 4. Hence the computation of λ by (6.1) is necessary.

The calculation of the wave drag force exerted on the dihedral or cone surface plays a very important role in the aeronautical applications. It can be shown what is given by the normalized formula (6.2). Knowing that ($k = 1, \varphi = \psi, \omega = 2, P = P_2$) for the dihedral and ($k = 2, \varphi = \theta_C, \omega = \pi, P = P_C$) for the cone.

$$D = \frac{Drag}{L l P_0} = \omega \tan^K(\varphi) \left(\frac{P}{P_{02}} \right) \left(\frac{P_{02}}{P_{01}} \right), \quad (6.2)$$

The values of P/P_{02} and P_{02}/P_{01} in (6.2) are respectively given by (5.3) and (4.1) respectively when $T = T_2$, $\rho = \rho_2$ and $P = P_2$, $\rho_0 = \rho_{02}$ and $P = P_{02}$ for the dihedral, and when $T = T_C$, $\rho = \rho_C$ and $P = P_C$, $\rho_0 = \rho_{02}$ for the cone. While φ and P_0 are among the data of our study. We have $P_0 = P_{01}$.

The C_x , C_y and C_m , and supersonic flow calculation for a given pointed airfoil is a fourth example for our applications. In Figure 5, a non-symmetrical curved pointed airfoil has been presented in an *HT* supersonic flow and under α , M_1 and T_0 data. The extrados and the intrados airfoil equations $f_E(x)$ and $f_I(x)$ are given by the relations (6.3) and (6.4).

$$f_E(x) = \frac{x}{10 C} \left(1 - \frac{x}{C} \right) \left(1 - \frac{2x}{3 C} \right) \left(1 - \frac{x}{2 C} \right), \quad (6.3)$$

$$f_I(x) = - \frac{x}{20 C} \left(1 - \frac{x}{C} \right) \left(1 - \frac{3x}{4 C} \right) \left(1 - \frac{5x}{8 C} \right). \quad (6.4)$$

Where $0 = x/C = 1$.

7. RESULTS AND COMMENTS

The presentation of the results for each shock wave is preferred in separate figures for good illustration.

All figures contain 9 curves for 9 different selected gases. Curve 1 for the H_2 gas. Curve 2 for O_2 . Curve 3 for N_2 . Curve 4 for CO . Curve 5 CO_2 . Curve 6 for H_2O . Curve 7 for NH_3 . Curve 8 for CH_4 and the curve 9 for air.

The choice of 9 gases in this study is based to have 3 or 5 gases giving better choices compared to air, and 5 or 3 other gases giving a bad choice compared to air.

TABLE 2. Gas effect on ψ_{max} for *OSW* at $T_0 = 2000$ K.

N	Gas	$M_1 = 2.00$	$M_1 = 3.00$	$M_1 = 4.00$	$M_1 = 5.00$
1	H ₂	24.0451	35.5024	40.1863	42.4514
2	O ₂	24.9934	37.1373	42.1355	44.5402
3	N ₂	24.5553	36.3724	41.2163	43.5503
4	CO	24.6511	36.5123	41.3614	43.6912
5	CO ₂	27.7378	42.1727	48.4294	51.5240
6	H ₂ O	26.7619	40.1466	45.6516	48.2529
7	NH ₃	28.7320	43.8759	50.3417	53.3940
8	CH ₄	29.8985	46.2645	53.6026	57.2007
9	Air	24.6714	36.7154	41.0122	43.8995

TABLE 3. Gas effect on ψ_{max} for *OSW* when $M_1 = 3.00$.

N	Gas	$T_0 = 298$	$T_0 = 1000$ K	$T_0 = 2000$ K	$T_0 = 3000$ K
1	H ₂	34.0020	34.3722	35.5024	36.5497
2	O ₂	34.1182	35.8017	37.1373	38.2496
3	N ₂	33.0729	35.1175	36.3724	37.4687
4	CO	33.7975	35.1575	36.5123	37.6854
5	CO ₂	38.6914	40.7564	42.1727	43.1916
6	H ₂ O	35.8732	37.8834	40.1466	41.8194
7	NH ₃	38.1136	40.7382	43.8759	45.6144
8	CH ₄	39.8452	42.9084	46.2645	47.7252
9	Air	33.9774	35.1931	36.7154	37.7759

In the figures of *OSW* and *CSW*, we presented the variation of all parameters only for the case of the *WS*, except the parameters β , ψ , M_2 and M_C , where we added the variation for the *SS*.

The commentary of the results was left at the end of the presentation of all *OSW* and *CSW* figures to avoid repetition of comments.

It may even be noted in all the presented figures that the arrangement of the numbers of the 9 curves of the selected gases for maximum (or minimum) to minimum (or maximum) value, in each figure and tables, is the same for all parameters and for all *OSW* and *CSW*. It is given by this order 1, 3, 4, 9, 2, 6, 5, 7 and 8.

For comparison, the results for air (curve 9) of *HT* model can be found in [2, 3, 13, 16, 17, 22, 25–27] for *OSW* and in [12] for *CSW*. It should be noted that the *HT* model presented in these references is different from the one presented in this study according to the modifications discussed in the introduction.

For *PG* model, the results can be found in [1] for *NSW* and *OSW* and in [18, 19, 24] for *CSW* also for comparison.

7.1. Results for *OSW*

Table 2 shows the gas and M_1 effect on the ψ_{Max} to have an attached *OSW* when $T_0 = 2000$ K.

Table 3 shows the gas and T_0 effect on the ψ_{Max} to have an attached *OSW* when $M_1 = 3.00$.

Table 4 shows the gas and ψ effect on the minimum value of M_1 to have an attached *OSW* when $T_0 = 2000$ K.

Table 5 shows the gas and T_0 effect on the minimum value of M_1 to have an attached *OSW* when $\psi = 20^\circ$.

Table 6 shows an example of gas effect on the aerodynamic coefficients of the airfoil of Figure 5 only for weak *OSW* when $M_1 = 3.00$, $T_0 = 2000$ K and $\alpha = 4^\circ$.

Table 7 represents an example of calculation of M , σ and β in the different isentropic flow zones after the reflection of a Weak *OSW* when $M_1 = 3.00$, $T_0 = 2000$ K and $\psi = 10^\circ$. Other parameters such as T_2/T_1 , ρ_2/ρ_1 ,

TABLE 4. Gas effect on M_{Lim} for *OSW* at $T_0 = 2000$ K.

N	Gas	$\psi = 10^\circ$	$\psi = 20^\circ$	$\psi = 30^\circ$	$\psi = 40^\circ$
1	H ₂	1.4055	1.7964	2.4015	3.9413
2	O ₂	1.3953	1.7645	2.3057	3.4736
3	N ₂	1.3999	1.7787	2.3476	3.6659
4	CO	1.3987	1.7754	2.3387	3.6310
5	CO ₂	1.3705	1.6915	2.1083	2.7808
6	H ₂ O	1.3780	1.7140	2.1691	2.9822
7	NH ₃	1.3624	1.6697	2.0566	2.6484
8	CH ₄	1.3544	1.6477	2.0041	2.5133
9	Air	1.3975	1.7751	2.3325	3.5731

TABLE 5. Gas effect on M_{lim} for *OSW* when $\psi = 20^\circ$.

N	Gas	$T_0 = 298$	$T_0 = 1000$ K	$T_0 = 2000$ K	$T_0 = 3000$ K
1	H ₂	1.8451	1.8264	1.7964	1.7729
2	O ₂	1.8398	1.7932	1.7645	1.7435
3	N ₂	1.8407	1.8083	1.7787	1.7563
4	CO	1.8417	1.8070	1.7754	1.7521
5	CO ₂	1.7502	1.7104	1.6915	1.6792
6	H ₂ O	1.7948	1.7510	1.7140	1.6922
7	NH ₃	1.7462	1.7053	1.6697	1.6534
8	CH ₄	1.7182	1.6785	1.6477	1.6360
9	Air	1.8404	1.8028	1.7759	1.7507

TABLE 6. Gas effect on C_x , C_y , C_m and $\Delta S/R$ for the airfoil of Figure 5 when $M_1 = 3.00$, $T_0 = 2000$ K and $\alpha = 4^\circ$.

N	Gas	$C_x \times 10^4$	$C_y \times 10^2$	$C_m \times 10^2$	$\Delta S/R \times 10$	χ
1	H ₂	0.2928	0.2640	0.1340	0.1204	90.163
2	O ₂	0.2766	0.2490	0.1266	0.1137	90.021
3	N ₂	0.2844	0.2562	0.1302	0.1166	90.091
4	CO	0.2835	0.2554	0.1297	0.1155	90.088
5	CO ₂	0.2216	0.1986	0.1013	0.1005	89.620
6	H ₂ O	0.2472	0.2219	0.1130	0.1009	89.765
7	NH ₃	0.2051	0.1836	0.0936	0.0927	89.517
8	CH ₄	0.1773	0.1585	0.0809	0.0906	89.396
9	Air	0.2781	0.2505	0.1273	0.1151	90.075

P_2/P_1 , P_{02}/P_{01} , $\Delta S_{12}/R$ between zone 2 and 1 and T_2/T_0 , ρ_2/ρ_{02} , P_2/P_{02} in zone 2, as well as T_3/T_2 , ρ_3/ρ_2 , P_3/P_2 , P_{03}/P_{02} , $\Delta S_{23}/R$ between zones 3 and 2, and T_3/T_0 , ρ_3/ρ_{03} , P_3/P_{03} in zone 3, and T_3/T_1 , ρ_3/ρ_1 , P_3/P_1 , P_{03}/P_{01} , $\Delta S_{13}/R$ between zones 1 and 3, and T_4/T_3 , ρ_4/ρ_3 , P_4/P_3 , P_{04}/P_{03} , $\Delta S_{34}/R$ between zones 3 and 4, and T_4/T_0 , ρ_4/ρ_{04} , P_4/P_{04} in the zone 4, and T_4/T_1 , ρ_4/ρ_1 , P_4/P_1 , P_{04}/P_{01} , $\Delta S_{14}/R$ between zones 1 and 4 can be easily deduced directly according to the values of Table 7.

Figure 6 shows the gas effect on the variation of β , M_2 , T_2/T_1 , ρ_2/ρ_1 , P_2/P_1 , P_{02}/P_{01} and $\Delta S_{12}/R$ through a weak *OSW* versus of ψ for $M_1 = 3.00$ and $T_0 = 2000$ K. Only for β and M_2 was added the variation for the strong shock.

Figure 7 shows the effect of gas on the variation of the isentropic parameters T_2/T_0 , ρ_2/ρ_{02} and P_2/P_{02} after weak *OSW* versus ψ when $M_1 = 3.00$ and $T_0 = 2000$ K.

(a) : Variation of β versus ψ . (b) : Variation of M_2 versus ψ . (c) : Variation of T_2/T_1 versus ψ . (d) : Variation of ρ_2/ρ_1 versus ψ . (e) : Variation of P_2/P_1 versus ψ . (f) : Variation of P_{02}/P_{01} versus ψ . (g) : Variation of $\Delta S_{12}/R$ versus ψ .FIGURE 6. Gas effect on the variation of OSW parameters *versus* ψ .

TABLE 7. Gas effect on the *OSW* reflection parameters of figure 4 when $M_1 = 3.00$, $\psi = 10^\circ$ and $T_0 = 2000$ K.

N	Gas	Zone 2		Zone 3		Zone 4	
		$\beta_2(^{\circ})$	M_2	$\beta_3(^{\circ})$	M_3	$\beta_4(^{\circ})$	M_4
1	H ₂	27.230	2.528	31.340	2.133	36.600	1.784
2	O ₂	27.016	2.566	30.720	2.194	35.385	1.861
3	N ₂	27.114	2.549	31.003	2.166	35.935	1.825
4	CO	27.099	2.551	30.959	2.170	35.845	1.831
5	CO ₂	26.474	2.669	29.158	2.366	32.433	2.082
6	H ₂ O	26.690	2.626	29.781	2.294	33.571	1.991
7	NH ₃	26.340	2.695	28.779	2.412	31.714	2.144
8	CH ₄	26.148	2.735	28.229	2.482	30.718	2.238
9	Air	27.059	2.559	30.837	2.183	35.620	1.845

FIGURE 7. Gas effect on the variation of the isentropic parameters behind *OSW* versus ψ .

Figure 8 shows the gas effect on the variation of the adaptation distance λ of a 2D air intake as a function of ψ for weak *OSW* when $M_1 = 3.00$ and $T_0 = 2000$ K.

Figure 9 shows the gas effect on the variation of the wave drag D on the dihedral as a function of ψ for weak *OSW* when $M_1 = 3.00$ and $T_0 = 2000$ K.

Figure 10 shows the gas effect on the variation of β , M_2 , T_2/T_1 , ρ_2/ρ_1 , P_2/P_1 , P_{02}/P_{01} and $\Delta S_{12}/R$ through a weak *OSW* versus of M_1 for $\psi = 20^\circ$ and $T_0 = 2000$ K. Only for β and M_2 was added the variation for the strong shock.

FIGURE 8. Gas effect on the variation of λ versus ψ . FIGURE 9. Gas effect on the variation of D versus ψ .

Figure 11 shows the effect of gas on the variation of the isentropic parameters T_2/T_0 , ρ_2/ρ_{02} and P_2/P_{02} after weak *OSW* versus M_1 when $\psi = 20^\circ$ and $T_0 = 2000$ K.

Figure 12 shows the gas effect on the variation of the adaptation distance λ of a 2D air intake as a function of M_1 for weak *OSW* when $\psi = 20^\circ$ and $T_0 = 2000$ K.

Figure 13 shows the gas effect on the variation of the wave drag D on the dihedral as a function of M_1 for weak *OSW* when $\psi = 20^\circ$ and $T_0 = 2000$ K.

7.2. Results for *CSW*

Table 8 shows the gas and M_1 effect on the ψ_{Max} to have an attached *CSW* when $T_0 = 2000$ K.

Table 9 shows the gas and T_0 effect on the ψ_{Max} to have an attached *CSW* when $M_1 = 3.00$.

Table 10 shows the gas and ψ effect on the minimum value of M_1 to have an attached *CSW* when $T_0 = 2000$ K.

Table 5 shows the gas and T_0 effect on the minimum value of M_1 to have an attached *CSW* when $\psi = 20^\circ$.

Figure 14 shows the gas effect on the variation of β , ψ , M_2 , T_2/T_1 , ρ_2/ρ_1 , P_2/P_1 , P_{02}/P_{01} and $\Delta S_{12}/R$ through a weak *CSW* versus of θC for $M_1 = 3.00$ and $T_0 = 2000$ K. Only for β , ψ and M_2 was added the variation for the strong shock.

Figure 15 shows the effect of gas on the variation of the isentropic parameters T_2/T_0 , ρ_2/ρ_{02} and P_2/P_{02} after weak *CSW* versus θC when $M_1 = 3.00$ and $T_0 = 2000$ K.

Figure 16 shows the effect of gas on the variation of the isentropic parameters M_C , T_C/T_0 , ρ_C/ρ_{02} and P_C/P_{02} on the cone surface for weak *CSW* versus θC when $M_1 = 3.00$ and $T_0 = 2000$ K. Only for M_C was added the variation for strong shock.

(a) : Variation of β versus M_1 . (b) : Variation of M_2 versus M_1 . (c) : Variation of T_2/T_1 versus M_1 . (d) : Variation of ρ_2/ρ_1 versus M_1 . (e) : Variation of P_2/P_1 versus M_1 . (f) : Variation of P_{02}/P_{01} versus M_1 . (g) : Variation of $\Delta S_{12}/R$ versus M_1 .FIGURE 10. Gas effect on the variation of *OSW* parameters versus M_1 .

FIGURE 11. Gas effect on the variation of the isentropic parameters behind *OSW* versus M_1 .

FIGURE 12. Gas effect on the variation of λ versus M_1 .

Figure 17 shows the gas effect on the variation of the adaptation distance λ of an axially symmetrical air intake as a function of θC for weak *CSW* when $M_1 = 3.00$ and $T_0 = 2000$ K.

Figure 18 shows the gas effect on the variation of the wave drag D on the cone as a function of θC for weak *CSW* when $M_1 = 3.00$ and $T_0 = 2000$ K.

FIGURE 13. Gas effect on the variation of D versus M_1 .TABLE 8. Gas effect on ψ_{max} for CSW at $T_0 = 2000$ K.

N	Gas	$M_1 = 2.00$	$M_1 = 3.00$	$M_1 = 4.00$	$M_1 = 5.00$
1	H ₂	42.1447	51.2780	54.8117	56.4785
2	O ₂	42.5895	51.8591	55.4671	57.1827
3	N ₂	42.1870	51.2433	54.7503	56.4203
4	CO	42.2762	51.3571	54.8651	56.5309
5	CO ₂	45.0537	55.8338	60.2896	62.4749
6	H ₂ O	44.1917	54.2511	58.1804	60.0199
7	NH ₃	45.9271	57.1472	61.7199	63.8596
8	CH ₄	46.9346	58.9735	64.1480	66.6659
9	Air	42.2863	51.5031	55.0319	56.6616

TABLE 9. Gas effect on ψ_{max} for CSW when $M_1 = 3.00$.

N	Gas	$T_0 = 298$	$T_0 = 1000$ K	$T_0 = 2000$ K	$T_0 = 3000$ K
1	H ₂	48.9344	49.6048	51.2880	52.2598
2	O ₂	50.2401	50.7721	51.8591	52.7508
3	N ₂	49.0980	49.4853	51.2433	52.1278
4	CO	49.0769	50.2192	51.3571	52.3015
5	CO ₂	53.1021	54.6856	55.8338	56.6229
6	H ₂ O	50.8301	52.4581	54.2511	55.5577
7	NH ₃	53.3412	54.7138	57.1472	58.4789
8	CH ₄	54.7541	56.3990	58.9735	60.0808
9	Air	49.2644	50.2367	51.5031	52.6540

Figure 19 shows the gas effect on the variation of β , ψ , M_2 , T_2/T_1 , ρ_2/ρ_1 , P_2/P_1 , P_{02}/P_{01} and $\Delta S_{12}/R$ through a weak CSW versus of M_1 for $\psi = 20^\circ$ and $T_0 = 2000$ K. Only for β , ψ and M_2 was added the variation for the strong shock.

Figure 20 shows the effect of gas on the variation of the isentropic parameters T_2/T_0 , ρ_2/ρ_{02} and P_2/P_{02} after weak CSW versus M_1 when $\psi = 20^\circ$ and $T_0 = 2000$ K.

Figure 21 shows the effect of gas on the variation of the isentropic parameters M_C , T_C/T_0 , ρ_C/ρ_{02} and P_C/P_{02} on the cone surface for weak CSW versus M_1 when $\psi = 20^\circ$ and $T_0 = 2000$ K. Only for M_C was added the variation for strong shock.

Figure 22 shows the gas effect on the variation of the adaptation distance λ of an axially symmetrical air intake as a function of M_1 for weak CSW when $\psi = 20^\circ$ and $T_0 = 2000$ K.

(a) : Variation of β versus θ_C . (b) : Variation of ψ versus θ_C . (c) : Variation of M_2 versus θ_C . (d) : Variation of T_2/T_1 versus θ_C . (e) : Variation of ρ_2/ρ_1 versus θ_C . (f) : Variation of P_2/P_1 versus θ_C . (g) : Variation of P_{02}/P_{01} versus θ_C . (h) : Variation of $\Delta S_{12}/R$ versus θ_C .FIGURE 14. Gas effect on the variation of *CSW* parameters versus θ_C .

FIGURE 15. Gas effect on the variation of isentropic parameters behind *CSW* versus θ_C .TABLE 10. Gas effect on M_{Lim} for *CSW* at $T_0 = 2000$ K.

N	Gas	$\psi = 10^\circ$	$\psi = 20^\circ$	$\psi = 30^\circ$	$\psi = 40^\circ$
1	H ₂	1.0508	1.2102	1.4503	1.8678
2	O ₂	1.0498	1.1982	1.4438	1.8470
3	N ₂	1.0512	1.2117	1.4512	1.8696
4	CO	1.0506	1.2078	1.4486	1.8629
5	CO ₂	1.0471	1.1865	1.4095	1.7491
6	H ₂ O	1.0479	1.1899	1.4198	1.7788
7	NH ₃	1.0462	1.1825	1.3985	1.7207
8	CH ₄	1.0453	1.1787	1.3878	1.6925
9	Air	1.0503	1.2004	1.4477	1.8618

Figure 23 shows the gas effect on the variation of the wave drag D on the cone as a function of M_1 for weak *CSW* when $\psi = 20^\circ$ and $T_0 = 2000$ K.

7.3. Comments

It should first be noted that whatever the used gas, there is its influence on all the physical parameters for the *OSW* and *CSW* according to all the presented figures in this work. Generally we are interested in *WS* for *OSW* and *CSW*. The *NSW* is of *SS* type.

(a) : Variation of M_C versus θ_C . (b) : Variation of T_C/T_0 versus θ_C . (c) : Variation of ρ_C/ρ_{02} versus θ_C . (d) : Variation of P_C/P_{02} versus θ_C .FIGURE 16. Gas effect on the variation of isentropic parameters on the cône surface *versus* θ_C . FIGURE 17. Gas effect on the variation of λ *versus* θ_C .

Between *WS* and *SS*, the inverse case is deduced on the influence rate of the gas on the variation of the parameters M_2 , P_{02}/P_{01} , $\Delta S_{12}/R$, ρ_2/ρ_{02} and P_2/P_{02} of *OSW* and *CSW*, and for β and λ of *OSW* and *CSW*, and for ψ , ρ_C/ρ_{02} , P_C/P_{02} of *CSW*. While the same influence rate is observed for the parameters T_2/T_1 , ρ_2/ρ_1 , P_2/P_1 , T_2/T_0 of *OSW* and *CSW*, and for D of *OSW* and *CSW*, and for T_C/T_0 of *CSW*.

According to the gas dynamic, aeronautical and aerospace applications discussed in this paper, it can be said to choose the interested parameters for each application.

FIGURE 18. Gas effect on the variation of D versus θ_C .

According to the 9 chosen gases presented in the Figures 6–22 and 23, it is noted that the influence of the gases CH_4 , NH_3 , CO_2 , H_2O and O_2 is of opposite direction to the behavior and the influence of the gases H_2 , N_2 , CO with respect to air. We also notice that the O_2 , despite being different in air, its behavior is very close compared to air with an error which does not exceed 1% at the maximum. We also note that the maximum deviation from air is found for the gas CH_4 , then NH_3 , then CO_2 and H_2O . The smallest (closest) error is found for O_2 , although the 4 gases H_2 , O_2 , N_2 and CO give a very small deviation from air, which gives the possibility of replacing air with one of these 4 gases, giving a very small change in the state of the flow.

For drag force (Figs. 9, 13, 18 and 23), the CH_4 becomes the right choice since there is a decrease in D of the engine. The bad is H_2 . For a cruise flight, it is best to have the smallest possible D . While landing or taking off, it's best to have as much drag as possible. We know that there are other maneuvers to increase or decrease this drag. But gas also has its own influence.

The flow around a pointed airfoil is characterized by the birth of an *OSW* at the leading edge then a series of progressive compression or expansion through the upper and lower surfaces. If there is an increase in the inclination of the wall, then we will have a compression of the flow and this study will be used to determine the local parameters of the flow. If there is a decrease in the inclination of the wall compared to the upstream flow, we will have in this case an expansion and the parameters of the flow will be determined by the use of the Prandtl Meyer function at *HT* [6, 26]. To see the interest of the gas, we added the aerodynamic fineness ($\chi = C_y/C_x$) for each gas. For C_x , C_y and C_m presented in the Table 6 for the airfoil of Figure 5, we must see the aerodynamic fineness which is observed maximum for H_2 and minimum for CH_4 . In this case the right choice is H_2 . O_2 , N_2 , CO gases are also good choices. CO_2 , H_2O , NH_3 and CH_4 are bad choices. To make a comparison, we added the results for air which are presented in [26].

The values presented in Table 6 vary with M_1 , T_0 and α as well as the used gas. All of these and other parameters will be the goal of future work.

In all *OSW* and *CSW* figures results (figures and tables), and for $M_1 < 1.50$ approximately, the choice of such a gas propulsion poses no importance, since the setting through the shock are almost the same.

It should be noted that for *OSW* and *CSW*, we have the possibility of having a detached shock wave according to the Tables 2 and 3 for *OSW* and the Tables 8 and 9 for *CSW*. According to Figures 6, 7, 8 and 9 for *OSW* and Figures 15, 16, 17 and 18 for *CSW*, we have no solution depending on the model of the attached shock wave. We must solve the Euler equation to detect the detached shock wave. The precise value of M_1 between attached and detached shock is given in Tables 4 and 5 for the *OSW* and Tables 10 and 11 for the *CSW*.

In all *OSW* and *CSW* figures results (figures and tables), if $\varphi < 10^\circ$ approximately, the choice of gas also has none, since the curves are almost confused. We can calculate the errors between each curves and curve 9 for air.

On Table 7 and according to the Mach number obtained in the three zones 2, 3 and 4, we notice a remarkable degradation of the Mach number after each reflection. Consequently there is also disadvantaged degradation of the other physical parameters which make the engine unreliable for aerodynamic flight characteristics. So to

(a) : Variation of β versus M_1 . (b) : Variation of ψ versus M_1 . (c) : Variation of M_2 versus M_1 . (d) : Variation of T_2/T_1 versus M_1 . (e) : Variation of ρ_2/ρ_1 versus M_1 . (f) : Variation of P_2/P_1 versus M_1 . (g) : Variation of P_{02}/P_{01} versus M_1 . (h) : Variation of $\Delta S_{12}/R$ versus M_1 .FIGURE 19. Gas effect on the variation of *CSW* parameters versus M_1 .

FIGURE 20. Gas effect on the variation of the isentropic parameters behind *CSW* versus M_1 .TABLE 11. Gas effect on M_{lim} for *CSW* when $\psi = 20^\circ$.

N	Gas	$T_0 = 298$	$T_0 = 1000$ K	$T_0 = 2000$ K	$T_0 = 3000$ K
1	H ₂	1.2124	1.2080	1.2089	1.1993
2	O ₂	1.2107	1.2030	1.1982	1.1948
3	N ₂	1.2129	1.2093	1.2099	1.1998
4	CO	1.2119	1.2050	1.2079	1.1984
5	CO ₂	1.1943	1.1897	1.1865	1.1843
6	H ₂ O	1.2035	1.1960	1.1899	1.1863
7	NH ₃	1.1927	1.1885	1.1825	1.1797
8	CH ₄	1.1904	1.1839	1.1787	1.1766
9	Air	1.2111	1.2040	1.2004	1.1973

avoid this situation, it is necessary to adapt the intake according to Figure 3 and to move the central body by a distance λ according to the relation (6.1) for each gas, giving the result presented on Figures 8 and 12. The effect of gas remains the same for an *OSW* configuration discussed previously.

The temperature T_0 also influences the parameters of *OSW* and *CSW*. In this study, we took the presentation of the results for $T_0 = 2000$ K. We did not prefer to vary the value of T_0 in the form of a graph, except in Tables 3 and 5 for *OSW*, Tables 9 and 11 for *CSW* for aim to calculate ψ_{Max} and M_{Lim} respectively. It should be noted that our application is designed at *HT*. The *PG* model does not depend on T_0 , and our *HT* model becomes a generalization of the *PG* model presented in [2, 3, 13, 16, 17, 22, 25–27].

(a) : Variation of M_C versus M_1 . (b) : Variation of T_C/T_0 versus M_1 . (c) : Variation of ρ_C/ρ_{02} versus M_1 . (d) : Variation of P_C/P_{02} versus M_1 .FIGURE 21. Gas effect on the variation of isentropic parameters on the cône surface *versus* M_1 . FIGURE 22. Gas effect on the variation of λ *versus* M_1 .

Due to the spread of waste and its combustion in the world with the increasing density for several years, the random cutting of trees that is harmful to the environment and the expectations that will occur in the future, we expect that the basic composition of the air will not return like that composed according to the Table 1 [4, 10, 14, 20, 21, 32]. For this reason it is necessary to take samples of the air in order to correct the thermodynamics of air in ambient medium in order to determine a correction at $C_P(T)$ and R .

We can use a gas instead of air having better performance compared to air after studying this gas according to our work for applications of aerodynamics (gas dynamic) external by blowing gas at the nose of the obstacle

FIGURE 23. Gas effect on the variation of D versus M_1 .

with a suitable flow rate making it possible to influence the inclination of the shock and consequently on all the parameters of shock, which gives us an gas dynamic acceleration or deceleration without mechanical activation.

The case of dissociation can be studied in order to see the possibility of finding results in hypersonic regime. In this case it is necessary to calculate the specific heat at constant pressure of the atoms constituting the dissociated gas according to the relation of the dissociation and to inject the resulting formula in the numerical program.

8. CONCLUSION

This work allows us to study the effect of use of the propellant gases on the thermodynamic parameters through an *OSW* and *CSW*. Can be drawn from the following conclusions:

- Numerical computation program can do any gas found in nature. It should be added in this case the variation of the specific heat $C_P(T)$ and the gas constant R even calculation of the function $H(T)$.
- The $C_P(T)$ function, R and $\gamma(T)$ ratio characterize the calorific value of the gas and mainly affects all parameters across the shock.
- The famous relation between (β, φ, M_1) at *HT* for *OSW* and *CSW* has been developed implicitly and numerically with great precision.
- The stagnation temperature and the Mach number also affect all parameters through the shock.
- The higher the ratio γ of the gas is high, the higher M_2 is very small also affects all other parameters. The jump of entropy in this case is big enough yet interpreted by the high load losses affecting for example the sound of the shock wave.
- The higher the ratio $\gamma(T)$ of the gases, the smaller the shock parameters. This type of gas is called by cold gas, this is the case for H_2 , O_2 , N_2 , CO and air.
- The *PG* model at low temperature considered for constant C_P becomes a particular case of our study at *HT*.
- It is recommended to take air samples every some years to correct the thermodynamics of the air in terms of $C_P(T)$ and R .
- We can use a gas instead of air having better performance compared to air after studying this gas according to our work for applications of aerodynamics (gas dynamic) external by blowing gas at the nose of the obstacle with a suitable flow rate making it possible to influence the inclination of the shock and consequently on all the parameters of shock.
- If we keep the same common parameter between gas and air and especially drag force, the use of another gas instead of air will improve or de-improve the use of this gas. Among the gases chosen, the use of H_2 , N_2 , CO by increasing M_2 , D and the use of gases CH_4 , NH_3 , CO_2 and H_2O will degrade M_2 and D .

- At low T and M , the difference in results between the different gases can be neglected, which shows the possibility of using any gas instead of air. We can go up to $M_1 = 1.50$ regardless of T_0 and φ , or $T_0 < 240$ K regardless of M_1 and φ , or $\varphi < 10^\circ$ regardless of T_0 and M_1 .
- The use of gases such as CH_4 , NH_3 , H_2O and CO_2 degrades M_2 compared to that of air. While H_2 , N_2 and CO increases M_2 compared to air, especially when T_0 is high.
- It is recommended to take samples of air from the friendly medium every certain year in order to correct the thermodynamics of the air in terms of $C_P(T)$ and R and chemical composition.
- The case of dissociation can be studied in order to see the possibility of finding results in hypersonic regime. In this case it is necessary to calculate the specific heat at constant pressure of the atoms constituting the dissociated gas according to the relation of the dissociation and to inject the resulting formula in the numerical program.

As perspectives, we can study our problem by the use of CFD or even make experimental tests by supersonic wind tunnel.

We can use the present work to study the effect of gas propulsion at HT to calculating the detached shock around a blunt body or pointed bodies with large wall inclination, or low flow Mach number, and to find the aerodynamic and gas dynamics coefficients and shock parameters.

NOMENCLATURE

C_P	Specific heat at constant pressure
a	Sound velocity.
H	Enthalpy.
M	Mach number.
θ	Polar radius deviation for CSW
θC	Cone deviation
P	Static pressure
R	Thermodynamic constant of gas
T	Temperature
V	Gas velocity
r	Polar ray
γ	Specific heats ratio
ρ	Density
a', b', c'	Coefficients of $C_P(T)$. See equation (1.1)
ψ	Dihedron deviation
β	Shock wave deviation
L	Longitudinal length of the dihedral
l	Unit of dihedral depth
ΔS	Entropy jump
h	Half height of the air intake nacelle
C	Airfoil chord
d	Adaptation distance of a supersonic air intake
λ	Standardized air intake adaptation distance
σ	Shock wave reflection with respect to the horizontal wall
D	Drag of the dihedral
HT	High Temperature
PG	Perfect Gas
NSW	Normal Shock Wave

OSW	Oblique Shock Wave
CSW	Conical Shock wave
SS	Strong Shock
WS	Weak Shock
$f_E(x)$	Equation of the Extrados airfoil
$f_I(x)$	Equation of the Intrados airfoil
α	Incidence angle of an airfoil
C_x	Drag coefficient of the airfoil
C_y	Lift coefficient of the airfoil
C_m	Pitching moment coefficient of the airfoil
k	$k = 1$ for dihedron and $k = 2$ for the cone
φ	$\varphi = \psi$ for dihedron and $\varphi = \theta C$ for the cone
ω	$\omega = 2$ for dihedron and $\omega = \pi$ for cone
χ	Aerodynamic finesse of an airfoil
CFD	Computational Fluid Dynamic

Indices

0_1	Stagnation conditions before the shock
0_2	Stagnation conditions behind the shock
0	Stagnation condition
1	Parameter just before the shock wave
2	Parameter just after the shock wave
3	Parameter in the zone 3 of Figure 5
4	Parameter in the zone 4 of the Figure 5
S	Shock
C	Parameter on the cone surface
r	In radial direction
θ	According to the transverse direction
Max	Maximum
Lim	Lower limit
a	Ambient condition

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