

MATHEMATICAL MODELING OF APHRON DRILLING NANOFLUID DRIVEN BY ELECTROOSMOTICALLY MODULATED PERISTALSIS THROUGH A PIPE

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Abstract. This analysis is conducted for a theoretical examination of the fluid flow characteristics and heat transferred by the nanoparticle-enhanced drilling muds flowing through drilling pipes under various physical conditions. Here, an important type of drilling fluid called Aphron drilling fluid is under consideration which is very effective for drilling in depleted regions. The rheological characteristics of the drilling fluid are predicted by Herschel-Bulkley fluid model. The fluid flow is driven by peristaltic pumping which is further aided by electroosmosis. The zinc oxide nanoparticles are dispersed in the aphron drilling fluid to prepare the nanofluid. The administering set of equations is simplified under the lubrication approach and the closed-form solutions are obtained for velocity and pressure gradient force. However, numerical solutions are executed for the temperature of nanofluid through built-in routine `bvp4c` of MATLAB. Fluid flow characteristics are analyzed for variation in physical conditions through graphical results. The outcomes of this study reveal that velocity profile substantially rises for application of forwarding electric field and temperature profile significantly decays in this case. An increment in temperature difference raises the magnitude of the Nusselt number. Furthermore, the nanoparticle volume fraction contributes to fluid acceleration and thermal conductivity of the drilling fluid.

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1. INTRODUCTION

Ever since an important observation of water flow under the action of the externally applied electric field has been presented by Ferdinand Friedrich Reuss about two centuries ago, microfluidics has gained attention in theoretical and experimental investigations. The noteworthy advantage of this observation is that the flow can be established without the involvement of any mechanical parts. The mechanism involved in such types of flows is termed as electrokinetics. Electrokinetic transport is based on the principle involving an external electric field and a charged surface whose charge is balanced by a layer of counter-ions which results in the creation of a well-known electric double layer (EDL) [15]. The ions confined in the interface are stationary however outer layer contains free ions which are set into motion upon implementation of an electric field. As a result, a net motion is generated which is known as electroosmosis.

Keywords and phrases: Aphron Drilling nanofluid, Electroosmosis, Herschel-Bulkley model, `bvp4c`, Peristalsis.

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Although this concept was reported in the 19th century, the phenomenon is considered as the main flow mechanism in the end decades of the 20th century after the fabrication of lab-on-chip devices and after that, microfluidics turned to be an emerging field of research in energy and biotechnology. The distribution of electric potential in the capillary under the approximation of small zeta potential was explored by [26]. A very basic modeling of electroosmotic flows by considering the Joule heating effect was presented by Xuan *et al.* [34]. Later on, multiple types of electroosmosis such as traveling wave electroosmosis [23] and induced charge electroosmosis [29] were reported. Nowadays, promising applications of electroosmosis can be found in various pumping mechanisms such as membrane [31] and peristaltic pumping [3]. The idea of combining electroosmosis with a peristaltic mechanism was presented by Chakraborty [10]. He got the inspiration for this modification from the fact that a large number of biological conduits follow the mechanism of peristalsis for their operation and the human body contains different types of electrolytes in order to regulate the functions of various organs. So, these two phenomena can be combined to achieve better control of fluid flows. This model can also be extended to study various nanoparticle enhanced physiological fluids which are used in various medical processes such as destroying cancer cells with a very small risk of damaging the healthy cells and targeted drug delivery etc. Recently, Akram *et al.* [4] studied the transport of blood-based graphene nanoparticles through combined electroosmosis and peristaltic mechanism. These nanoparticles can be potential drug carriers, can act as cancer therapy agents, in gastrointestinal endoscopy, and as imaging molecules. Vaidya *et al.* [32] performed an investigation on the peristaltic propulsion of MHD Jefferey fluid through a vertical channel and also studied the entropy generation phenomenon during the fluid flow. They have considered the effect of homogeneous-heterogeneous reactions and used the perturbation technique for the solutions to the problem. The flow of the blood through an artery having multiple stenoses of both symmetric and non-symmetric shapes is studied by Zidan *et al.* [36]. Several theoretical studies were also conducted to discuss the flow motions driven by combined electroosmosis and peristaltic pumping [2, 5, 6, 17, 21, 22, 27].

Aphron-based drilling fluids are the complex combination of solids, base fluids, polymers, some surfactants, and stabilizers to generate and stabilize the microbubbles called Aphron [28]. Such fluids are pumped into the drill bit during the drilling process for lubrication, cooling, and cleaning purposes. The complex nature of such fluids can be described by non-Newtonian rheological models such as the Bingham plastic model, Herschel-Bulkley model, power-law model, etc. Ehsan *et al.* [12] investigate the rheological characteristics of aphron-based drilling fluids and reported that the rheology of such fluids can be more accurately described by Herschel-Bulkley fluid model. Some investigations in this direction revealed that aphron drilling fluids can be efficiently utilized in oil fields for drilling in the depleted formation with a very small loss of drilling fluids due to the presence of stabilized aphron [8, 14, 25]. The application of aphron drilling fluid in reducing the formation damages during the drilling of horizontal wells is reported in [9].

Over the last few decades, nanotechnology has provided us with an efficient solution to overcome the increasing demands of much more efficient and cost-effective heat transfer fluids to encounter the cooling challenges in electronic devices, energy supply systems, and biomedical domains. The thermal performance of non-metallic fluids can be significantly improved by inserting some nanosized metallic particles of size less than or equal to 100 nm [11]. The main purpose of drilling fluids is lubrication and cooling of the drill bit. Nanotechnology can be used to develop drilling muds with improved rheological and thermal properties, and hence enhancing the efficiency of drilling muds to a larger extent [30, 35]. The effect of nanoparticles on the stability of drilling fluids is investigated by Agarwal *et al.* [1] and they found that nanoparticles can stabilize the drilling fluids without affecting the rheological characteristics of the fluid. Few more investigations in this direction are reported in [19, 20].

After reviewing the literature in-depth, it has been found that no investigation has been carried out on Electroosmotically modulated peristaltic propulsion of drilling nanofluid through a pipe. Our foremost aim is to develop a theoretical model for nanoparticle-enhanced aphron drilling fluid by incorporating the Herschel-Bulkley fluid model and Buongiorno nanofluid model. The inherent effects of Joule heating and viscous dissipation are retained in this investigation. Moreover, experimentally calculated rheological parameters of Herschel-Bulkley fluid for aphron-based drilling fluid are used to get the accurate behavior of fluid flow properties. The momentum equation is solved directly for the velocity field and temperature solutions are obtained

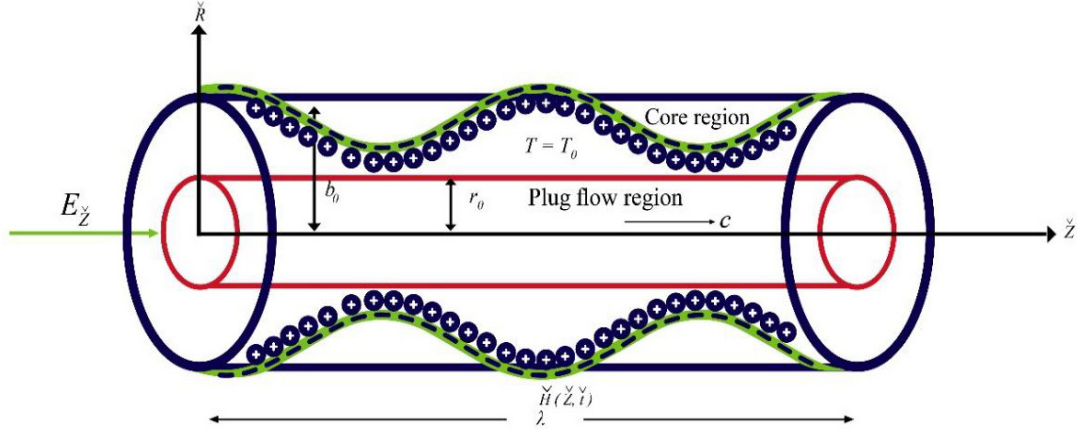


FIGURE 1. Schematic representation of the problem.

by the bvp4c package of MATLAB. The technique demands the reduction of constitutive equations into a set of 1st order ODEs and utilizes the principles of the finite difference scheme to calculate the solutions. A brief discussion on the simulated results of the investigation is presented. This mathematical model can help in understanding the effect of various parameters on the transport of drilling fluids in order to optimize the wellbore cleaning process during the drilling process and improve their heat transport tendency. This model can be extended for investigating the other types of drilling mud such as oil-based muds, synthetic-based drilling muds, etc. Further, in order to improve the cleaning and cooling of drill formations, this model can be used to compare the lubricating and heat transfer properties of various nanoparticles enhanced drilling muds.

2. MODEL FORMULATION

2.1. Flow regime

In the present exploration, the electroosmotically aided peristaltic pumping of nanoparticles enhanced aphron-based drilling fluids through a pipe is considered. Aphron generation is basically the generation of microbubbles in order to reduce the drilling fluid losses in different drilling formations. Aphron can be generated by adding some stabilizers such as Xanthan gum in the water-based drilling muds (WBM). Here a theoretical investigation is carried out to study the fluid flow behavior and the advancements in heat transfer properties of nanoparticles enhanced water-based drilling fluids (NWBM). The nanofluid is prepared by dispersal of zinc oxide nanoparticles of about 30nm diameter in the drilling fluid. The fluid flow is induced by the propulsion of the sinusoidal wave trains along the walls of the pipe which is further augmented by the process of electroosmosis. As drilling fluids contain a sufficient amount of salt [12] so aphron based drilling fluids can be considered as ionic liquids and electroosmotic velocity can be generated across the tube walls by the application of an electric field along the walls. The cylindrical coordinate system $(\tilde{R}, \tilde{Z}, \tilde{t})$ are adopted to formulate the mathematical model for the problem. The schematic diagram for fluid flow is exhibited in Figure 1 and mathematical form of deformed channel walls is described as:

$$\tilde{R} = \pm \tilde{H}(\tilde{Z}, \tilde{t}) = \pm \left(d + a \sin \left(\frac{2\pi}{\lambda} (\tilde{Z} - c\tilde{t}) \right) \right). \quad (2.1)$$

In which d designates the radius of the pipe, a is the amplitude, λ is the wavelength, and c is the speed of the peristaltic wave.

2.2. Governing equations and their scaling

In this article, a theoretical model is developed to study the lubrication and heat transfer characteristics of NWBM through a drilling pipe which is driven combinedly by electroosmosis and peristaltic pumping. Aphron-based drilling fluid exhibits viscoplastic behavior which can be accurately described by Herschel Bulkley fluid model. The parameters involved in Herschel-Bulkley fluid model are calculated experimentally in [12] for the drilling fluids under consideration and are utilized in this investigation. The nanofluid properties are studied through the modified Buongiorno nanofluid model and the thermal conductivity of drilling nanofluids is predicted by the Corcione model. The analysis is carried out in the presence of thermal radiation, Joule heating, and viscous dissipation. The walls of the pipe are assumed to be porous which are further assumed to be convectively heated.

The set of equations governing the flow is obtained after invoking the above-mentioned inferences as [16]:

$$\frac{\partial \check{U}}{\partial \check{R}} + \frac{\check{U}}{\check{R}} + \frac{\partial \check{W}}{\partial \check{Z}} = 0, \quad (2.2)$$

$$\rho_{nf} \left(\frac{\partial \check{U}}{\partial \check{t}} + \check{U} \frac{\partial \check{U}}{\partial \check{R}} + \check{W} \frac{\partial \check{U}}{\partial \check{Z}} \right) = -\frac{\partial \check{P}}{\partial \check{R}} + \frac{1}{\check{R}} \frac{\partial (\check{R} \check{\tau}_{\check{R}\check{R}})}{\partial \check{R}} + \frac{\partial (\check{\tau}_{\check{R}\check{Z}})}{\partial \check{Z}} - \frac{\check{\tau}_{\check{\theta}\check{\theta}}}{\check{R}}, \quad (2.3)$$

$$\rho_{nf} \left(\frac{\partial \check{W}}{\partial \check{t}} + \check{U} \frac{\partial \check{W}}{\partial \check{R}} + \check{W} \frac{\partial \check{W}}{\partial \check{Z}} \right) = -\frac{\partial \check{P}}{\partial \check{Z}} + \frac{1}{\check{R}} \frac{\partial (\check{R} \check{\tau}_{\check{R}\check{Z}})}{\partial \check{R}} + \frac{\partial (\check{\tau}_{\check{Z}\check{Z}})}{\partial \check{Z}} + \rho_e E_{\check{Z}}, \quad (2.4)$$

$$\begin{aligned} (\rho C_p)_{nf} \left(\frac{\partial \check{T}}{\partial \check{t}} + \check{U} \frac{\partial \check{T}}{\partial \check{R}} + \check{W} \frac{\partial \check{T}}{\partial \check{Z}} \right) &= k_{nf} \left(\frac{\partial^2 \check{T}}{\partial \check{R}^2} + \frac{1}{\check{R}} \frac{\partial \check{T}}{\partial \check{Z}} + \frac{\partial^2 \check{T}}{\partial \check{Z}^2} \right) + \tau \cdot L^t \\ &+ (\rho C_p)_s D_B (\nabla \check{\Phi} \cdot \nabla \check{T}) + \frac{D_T}{T_m} (\rho C_p)_s (\nabla \check{T} \cdot \nabla \check{T}) - \frac{\partial q_{\check{R}}}{\partial \check{R}} + \sigma_{nf} E_{\check{Z}}^2, \end{aligned} \quad (2.5)$$

$$\left(\frac{\partial \check{\Phi}}{\partial \check{t}} + \check{U} \frac{\partial \check{\Phi}}{\partial \check{R}} + \check{W} \frac{\partial \check{\Phi}}{\partial \check{Z}} \right) = \nabla \cdot \left(D_B \nabla \check{\Phi} + D_T \frac{\nabla \check{T}}{T_m} \right). \quad (2.6)$$

Here $q_{\check{R}}$ is the radiative heat flux which is calculated by Rosseland approximation given by [24]:

$$q_{\check{R}} = -\frac{16\sigma^*}{3\kappa^*} T_0^3 \frac{\partial \check{T}}{\partial \check{R}}, \quad (2.7)$$

with σ^* and κ^* being Stefan-Boltzmann constant and mean absorption coefficient respectively. Moreover,

$$L = \text{grad}(\vec{V}), \quad (2.8)$$

The other quantities involved in the above relations can be described as: ρ_{nf} is the nanofluid density, $\check{P}$ is the pressure force, $\check{U}$ and $\check{W}$ are the velocity components in $\check{R}$ and $\check{Z}$ directions respectively, ρ_e is the electric charge density, $\check{T}$ is the temperature of the fluid, τ is the stress tensor, $(\rho C_p)_{nf}$ and $(\rho C_p)_s$ are the specific heat capacity of nanofluid and solid particles respectively, $E_{\check{Z}}$ is the electric body force along the length of the pipe, D_B is the Brownian diffusion parameter, D_T denotes the thermophoretic parameter, and $\check{\Phi}$ shows the nanoparticle concentration.

The stress tensor for the Herschel-Bulkley fluid model after inserting Einstein's model for the viscosity of nanofluid is given by [33]:

$$\boldsymbol{\tau} = \tilde{\tau}_0 + \frac{K}{(1 - \phi_0)^{2.5}} (\dot{\gamma})^n, \quad (2.9)$$

where $\tilde{\tau}_0$ is the yield stress, K is the consistency index, and n represents the power-law index. Furthermore, $\dot{\gamma}$ is defined as:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \text{tr}(A_1^2)}, \quad (2.10)$$

$$A_1 = L + L^t, \quad (2.11)$$

in which A_1 represents the first Rivlin-Erickson tensor.

The physical properties of the nanofluid are calculated from the general mixture rule as:

$$\rho_{nf} = (1 - \phi_0) \rho_{bf} + \phi_0 \rho_s, \quad (2.12)$$

$$(\rho C_p)_{nf} = (1 - \phi_0) (\rho C_p)_{bf} + \phi_0 (\rho C_p)_s, \quad (2.13)$$

The Corcione model is described as:

$$\frac{k_{nf}}{k_{bf}} = \left(1 + 4.4 R^{0.4} Pr^{0.66} \left(\frac{T_0}{T_{fr}} \right)^{10} \left(\frac{k_s}{k_{bf}} \right)^{0.03} \phi_0^{0.66} \right) = k_r, \quad (2.14)$$

The electrical conductivity σ_{nf} of nanofluid is estimated by the Maxwell-Garnett model by the relation given as:

$$\sigma_{nf} = \sigma_{bf} \left(1 + \frac{\left(\frac{\sigma_s}{\sigma_{bf}} - 1 \right) \phi_0}{\left(\frac{\sigma_s}{\sigma_{bf}} + 2 \right) - \left(\frac{\sigma_s}{\sigma_{bf}} - 1 \right) \phi_0} \right). \quad (2.15)$$

where ϕ_0 specifies the volume fraction of the zinc oxide nanoparticles in drilling fluid, and the subscripts 's' and 'f' are utilized for the nanoparticle properties, and the base fluid properties respectively.

The electric potential generated by the electroosmotic flow is governed by the well-known Poisson equation as [4]:

$$\nabla^2 \check{E} = -\frac{\rho_e}{\varepsilon_0 \varepsilon_m}, \quad (2.16)$$

where E is the electric potential, ε_0 is the vacuum dielectric constant and ε_m is the relative permittivity constant for the fluid medium. The electric charge density ρ_e following the Boltzmann ionic distribution is expressed as:

$$\rho_e = -2ze n_0 \sinh \left(\frac{ezE}{k_B \hat{T}_{avg}} \right), \quad (2.17)$$

in which z represents the valence of the ionic species, e is the charge on the electron, k_B is the Boltzmann constant, $\hat{T}_{avg}$ is the average temperature and n_0 is the bulk concentration of ionic species.

In order to observe a steady-state fluid flow, the fluid model is transformed from a laboratory frame $(\check{R}, \check{Z}, \check{t})$ to wave frame $(\dot{r}, \dot{z})$ which travels with the same speed as that of the peristaltic wave. The transformation relations are given by:

$$\dot{z} = \check{Z} - c\check{t}, \quad \dot{r} = \check{R}, \quad \dot{u} = \check{U}, \quad \dot{w} = \check{W} - c, \quad \dot{p}(\dot{r}, \dot{z}) = \check{P}(\check{R}, \check{Z}, \check{t}). \quad (2.18)$$

The non-dimensional analysis of the problem is facilitated by introducing the following dimensionless quantities:

$$\begin{aligned} z &= \frac{\dot{z}}{\lambda}, \quad r = \frac{\dot{r}}{d}, \quad p = \frac{\dot{p}d^{n+1}}{Kc^n\lambda}, \quad u = \frac{\dot{u}}{c}, \quad w = \frac{\dot{w}}{c}, \quad h = \frac{\tilde{H}}{d} = 1 + \varepsilon \sin(2\pi z), \\ \varepsilon &= \frac{a}{d}, \quad Pr = \frac{K(C_p)_{bf}}{k_{bf}} \left(\frac{c}{d}\right)^{n-1}, \quad \delta = \frac{d}{\lambda}, \quad \theta = \frac{\check{T} - T_0}{T_1 - T_0}, \quad \Delta T = T_1 - T_0, \quad \tau_0 = \frac{\check{\tau}_0 d^n}{Kc^n}, \\ \Phi &= \frac{\check{\Phi} - \phi_0}{\phi_0}, \quad \check{E} = \frac{ezE}{k_B \hat{T}_{avg}}, \quad Re = \frac{\rho_{bf} cd}{K \left(\frac{c}{d}\right)^{n-1}}, \quad S = \frac{\sigma_f E_z^2 d^2}{k_{bf} \Delta T}, \quad \Psi = \frac{\check{\Psi}}{cd}, \\ U_{hs} &= -\frac{\varepsilon_0 \varepsilon_m k_B \hat{T}_{avg} E_z}{ezKc \left(\frac{c}{d}\right)^{n-1}}, \quad N_t = \frac{D_T \tau^* \rho_{bf} \Delta T}{\tilde{T}_m K \left(\frac{c}{d}\right)^{n-1}}, \quad N_b = \frac{D_B \tau^* \rho_{bf} \phi_0}{K \left(\frac{c}{d}\right)^{n-1}}, \\ \tau^* &= \frac{(\rho C_p)_s}{(\rho C_p)_{bf}}, \quad Ec = \frac{c^2}{(C_p)_{bf} \Delta T}, \quad Br = Ec Pr, \quad Rd = \frac{16\sigma^* T_0^3}{3k^* K \left(\frac{c}{d}\right)^{n-1} (C_p)_{bf}}, \\ u &= -\frac{\delta}{r} \Psi_z, \quad w = \frac{1}{r} \Psi_r. \end{aligned} \quad (2.19)$$

In the above equation, Re specifies the Reynolds number, N_t is the non-dimensional thermophoretic parameter, U_{hs} is the electroosmotic velocity parameter, Rd is the radiation parameter, Br is the Brinkmann number, Ec is the Eckert number, τ_0 is the dimensionless yield stress parameter, S is the Joule heating parameter, ΔT is the temperature difference, N_b is the Brownian diffusion parameter in dimensionless parameter, and θ and Φ are the dimensionless temperature and concentration parameters respectively.

With the help of equation (2.18)–(2.19) and adopting the lubrication approach, *i.e.*, the assumption of long wavelength and low Reynolds number, continuity equation (2.2) is identically satisfied, and equations (2.3)–(2.11) and equations (2.16)–(2.17) are simplified to:

$$\frac{\partial p}{\partial r} = 0, \quad (2.20)$$

$$\frac{1}{r} \frac{\partial (r \tau_{rz})}{\partial r} + U_{hs} \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial E}{\partial r} \right) \right) = -\frac{\partial p}{\partial z}, \quad (2.21)$$

$$K_r \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} \right) + A_2 Pr N_b \frac{\partial \theta}{\partial r} \frac{\partial \Phi}{\partial r} + A_2 Pr N_t \left(\frac{\partial \theta}{\partial r} \right)^2 + \sigma_r S + Br \tau_{rz} \left(\frac{\partial w}{\partial r} \right) + Pr Rd \frac{\partial^2 \theta}{\partial r^2} = 0, \quad (2.22)$$

$$\left(\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r}\right) + \frac{N_t}{N_b} \left(\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r}\right) = 0 \quad (2.23)$$

$$\tau_{rz} = \tau_0 + A \left(-\frac{\partial w}{\partial r}\right)^n, \quad (2.24)$$

$$\tau_{xx} = \tau_{rr} = 0, \quad (2.25)$$

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} = k^2 \sinh(E), \quad (2.26)$$

which, after using the Debye-Hückel linearization approximation, reduces to:

$$\frac{\partial^2 E}{\partial r^2} + \frac{1}{r} \frac{\partial E}{\partial r} = k^2 E. \quad (2.27)$$

equation (2.21) in its simplified form is still not tractable for the analytical solutions due to the involvement of electric body forces in the momentum equation. To overwhelm this difficulty, the electroosmotic velocity parameter can be introduced in the boundary condition for velocity as the electroosmotic slip parameter and electric body forces are completely eliminated from the momentum equation. This modification is well justified as it is already assumed under Debye-Hückel approximation that a very low zeta potential is established across electric double layer (EDL) and EDL thickness is very small as compared to pipe radius so, electroosmotic forces are effective in the vicinity of pipe walls only [13]. And hence in this way, the solution of equation (2.27) is not required here. Under this modification, momentum equation (2.21) and the corresponding boundary conditions are expressed as:

$$\frac{1}{r} \frac{\partial (r \tau_{rz})}{\partial r} = -\frac{\partial p}{\partial z}, \quad \text{for } \tau_{rz} \geq \tau_0 \quad (2.28)$$

$$\frac{\partial w}{\partial r} = 0 \quad \text{for } \tau_{rz} \leq \tau_0$$

$$w + \frac{\sqrt{Da}}{\alpha} \frac{\partial w}{\partial r} = U_{hs} - 1 \quad \text{at } r = h, \quad (2.29)$$

$$\tau_{rz} \text{ is finite at } r = 0.$$

Here Da designates the Darcy number and α is the velocity slip parameter.

The prescribed boundary conditions for temperature and nanoparticle concentration in the dimensionless form are:

$$\frac{\partial \theta}{\partial r} = 0, \quad \frac{\partial \Phi}{\partial r} = 0, \quad \text{at } r = 0, \quad (2.30)$$

$$\frac{\partial \theta}{\partial r} + Bi\theta = 0, \quad \frac{\partial \Phi}{\partial r} + \frac{N_t}{N_b} \frac{\partial \theta}{\partial r} = 0, \quad \text{at } r = h$$

where Bi represents the thermal Biot number.

Equation (2.29) subject to boundary conditions in equation (2.30) is solved for the velocity field as:

$$w = \frac{2^{-\frac{1}{n}} An \left(\frac{hP-2\tau_0}{A} \right)^{1+\frac{1}{n}}}{(1+n)P} - \frac{2^{-\frac{1}{n}} An \left(\frac{Pr-2\tau_0}{A} \right)^{1+\frac{1}{n}}}{(1+n)P} + \frac{2^{-\frac{1}{n}} \sqrt{Da} \left(\frac{hP-2\tau_0}{A} \right)^{\frac{1}{n}}}{\alpha} + U_{hs} - 1, \quad (2.31)$$

Here $P = -\frac{\partial p}{\partial z}$, $A = \frac{1}{(1-\phi_0)^{2.5}}$

The upper limit of the plug flow can be found by invoking the boundary condition $\frac{\partial w}{\partial r} = 0$ at $r = r_0$ and it is found to be $r_0 = \frac{\tau_0}{P}$.

Now imposing the boundary condition $\tau_{rz} = \tau_h$ at $r = h$ gives $P = \frac{\tau_h}{h}$. Hence, we get:

$$\frac{r_0}{h} = \frac{\tau_0}{\tau_h} = \tau, \quad \text{for } 0 < \tau < 1. \quad (2.32)$$

The plug flow velocity can be obtained by inserting $r = r_0$ in equation (2.31) and using the relation expressed in equation (2.32) as:

$$w_p = \frac{2^{-\frac{1}{n}} \left(-\frac{hP(-1+\tau)}{A} \right)^{\frac{1}{n}} (\sqrt{Da}(1+n) - hn(-1+\tau)\alpha)}{(1+n)\alpha} + U_{hs} - 1, \quad (2.33)$$

The stream function for the plug flow region is calculated by the integration of the relation $w_p = \frac{1}{r} \frac{\partial \psi_p}{\partial r}$ subject to the boundary condition $\psi_p = 0$ at $r = 0$, we get the following result:

$$\psi_p = \frac{r^2}{2} \left(\frac{2^{-\frac{1}{n}} \left(-\frac{hP(-1+\tau)}{A} \right)^{\frac{1}{n}} (\sqrt{Da}(1+n) - hn(-1+\tau)\alpha)}{(1+n)\alpha} + U_{hs} - 1 \right), \quad (2.34)$$

Now, integrating $w = \frac{1}{r} \frac{\partial \psi}{\partial r}$ using equation (2.31) and boundary condition $\psi = \psi_p$, the stream function is obtained as:

$$\begin{aligned} \psi = & \frac{1}{(1+n)\alpha} 2^{-\frac{1+n}{n}} \left(\frac{1}{(1+n(5+6n))P^3} (\sqrt{Da}(1+n)(1+2n)(1+3n)P^3 \right. \\ & \left. (r^2 \left(-\frac{hP(-1+\tau)}{A} \right)^{\frac{1}{n}} + (r-h\tau)(r+h\tau) \left(\frac{hP-2\tau_0}{A} \right)^{\frac{1}{n}}) + n\alpha(h \right. \\ & \left. (1+n(5+6n))P^3(-r^2(-1+\tau) \left(-\frac{hP(-1+\tau)}{A} \right)^{\frac{1}{n}} + (r-h\tau) \right. \\ & \left. (r+h\tau) \left(\frac{hP-2\tau_0}{A} \right)^{\frac{1}{n}}) - 2(1+n(5+6n))P^2(r-h\tau)(r+h\tau) \right. \\ & \left. \left(\frac{hP-2\tau_0}{A} \right)^{\frac{1}{n}} \tau_0 - 2n(Pr-2\tau_0)^2 \left(\frac{Pr-2\tau_0}{A} \right)^{\frac{1}{n}} (Pr+2nPr+2n\tau_0)\tau_0 \right. \\ & \left. + 2n(hP\tau-2\tau_0)^2 \left(\frac{hP\tau-2\tau_0}{A} \right)^{\frac{1}{n}} (hP\tau+2hnP\tau+2n\tau_0)) + 2^{\frac{1}{n}} \sqrt{Da} \right. \\ & \left. (1+n)(2r^2-h^2\tau^2)U_{hs}) + \frac{(2r^2-h^2\tau^2)(-\alpha+(-\sqrt{Da}+\alpha)U_{hs})}{2\alpha}, \end{aligned} \quad (2.35)$$

The volume flow rate throughout the cross-section of the pipe can be calculated from the relation:

$$q = 2 \int_0^{r_0} r w_p dr + \int_{r_0}^h r w dr, \quad (2.36)$$

$$q = h(2^{\frac{-1}{n}} (1 + 3n) \left(-2^{\frac{1}{n}} h (1 + n) (1 + 2n) P^2 \right) \alpha + h P^2 \left(-\frac{h P (-1 + \tau)}{A} \right)^{\frac{1}{n}} \quad (2.37)$$

$$\frac{\sqrt{Da} (1 + n) (1 + 2n) (1 + 3n) - h n (-1 + \tau + n (-3 + \tau + 2\tau^2 + 2n (-1 + \tau^3)) \alpha)}{(1 + n) (1 + 2n) (1 + 3n) P^2 \alpha} + h U_{hs},$$

The non-dimensional averaged flux is given by:

$$Q = \int_0^1 \int_0^h r (w - 1) dr dz = q + \int_0^1 h^2 dz = q + 1 + \frac{\varepsilon^2}{2}. \quad (2.38)$$

From equation (2.2), we can get an expression for the pressure gradient force as:

$$P = -\frac{dp}{dz} = -\frac{A \left(\frac{2^{\frac{1}{n}} (1 + n) (1 + 2n) (1 + 3n) \left(2q\alpha - \sqrt{Da} h^2 U_{hs} \right)}{h^2 \left(-\sqrt{Da} (1 + n) (1 + 2n) (1 + 3n) + h n (-1 + \tau + n (-3 + \tau + 2\tau^2 + 2n (-1 + \tau^3)) \alpha \right)} \right)}{h (-1 + \tau)}, \quad (2.39)$$

Now, the pressure rise across one wavelength is computed by the integration of pressure difference over one wavelength as:

$$\Delta P_\lambda = \int_0^1 -\frac{dp}{dz} dz, \quad (2.40)$$

and frictional force can be obtained as:

$$F = \int_0^1 h^2 \frac{dp}{dz} dz. \quad (2.41)$$

The dimensionless Nusselt number at the upper wall is computed by:

$$Nu = k_r \left(\frac{-\partial \theta}{\partial r} \right)_{r \rightarrow h}. \quad (2.42)$$

2.3. Solution methodology

Equations (2.22)–(2.23) along with boundary conditions given in equation (2.2) are nonlinear and coupled equations so, the boundary value problem cannot be tackled to acquire the closed-form solution expressions. Therefore, a built-in routine `bvp4c` in mathematical software MATLAB is opted to obtain accurate numerical results for the considered BVP. This numerical solver is based on the finite difference scheme. In this method, the nonlinear higher-order equations are transformed into a set of 1st order ODEs which are then executed for the required solution by setting a suitable step size. The obtained solutions for the temperature and Nusselt number are plotted for multiple values of various involved parameters.

TABLE 1. Thermophysical attributes of the zinc oxide nanoparticles, and Aphron drilling fluid [1, 18].

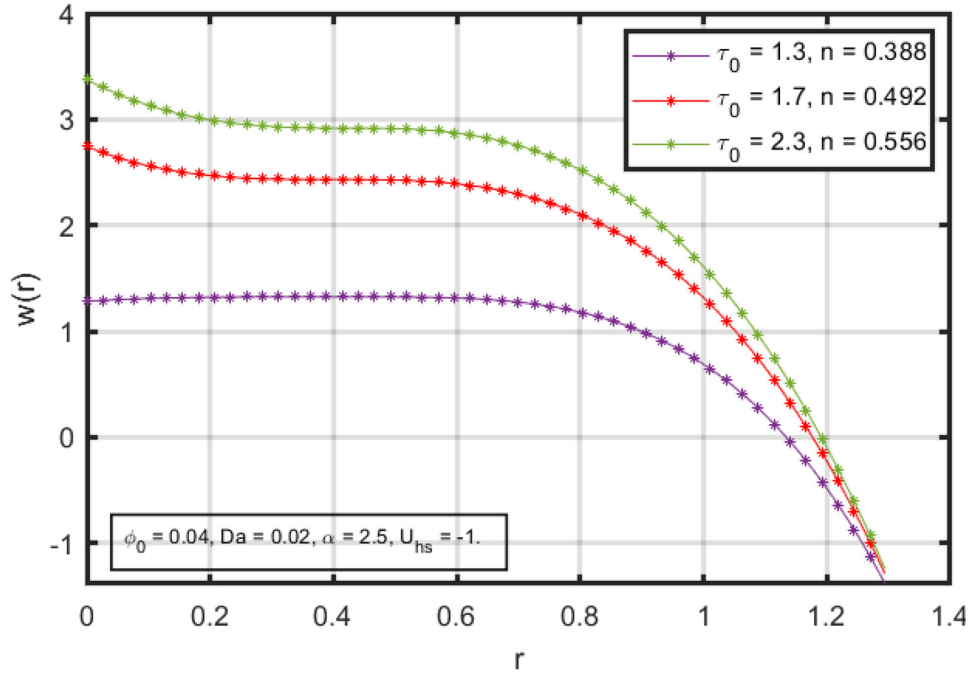
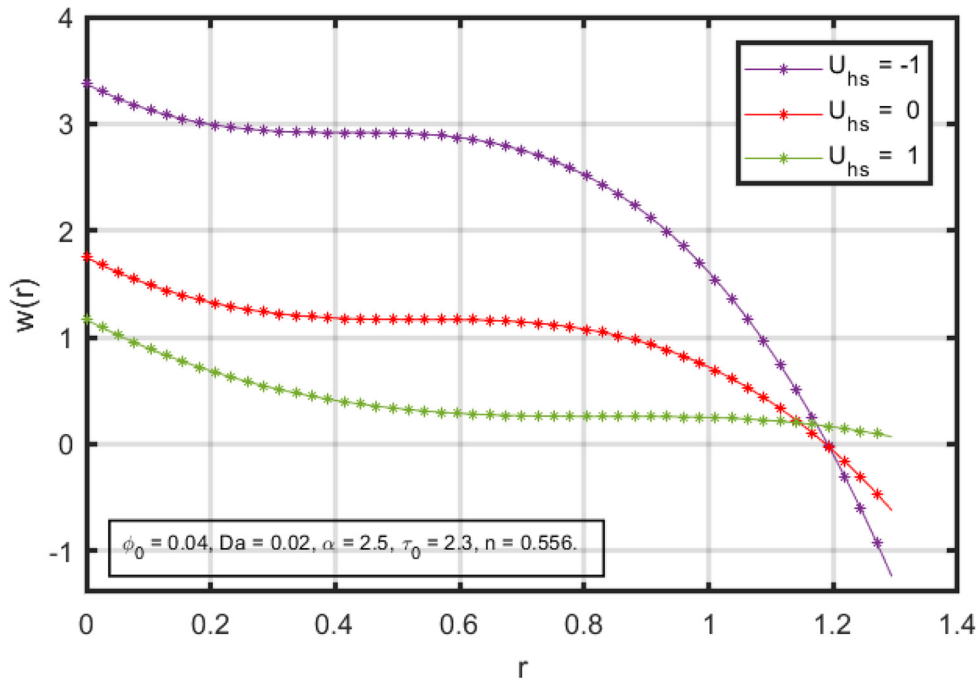
Physical properties	Zinc oxide	Aphron Drilling fluid
Specific heat capacity (J/Kg. K)	516	3628
Density (kg/m ³)	5600	1027
Thermal conductivity(W/mK)	29	0.5120
Particle diameter (d)	30 nm	----
Electric conductivity (S/m)	0.004	40.3

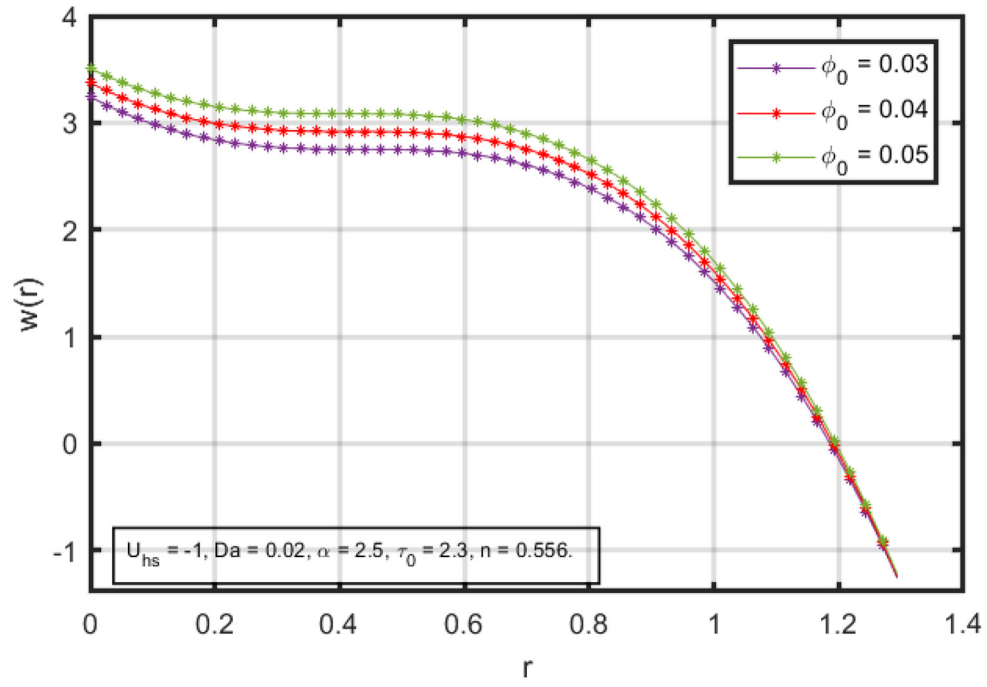
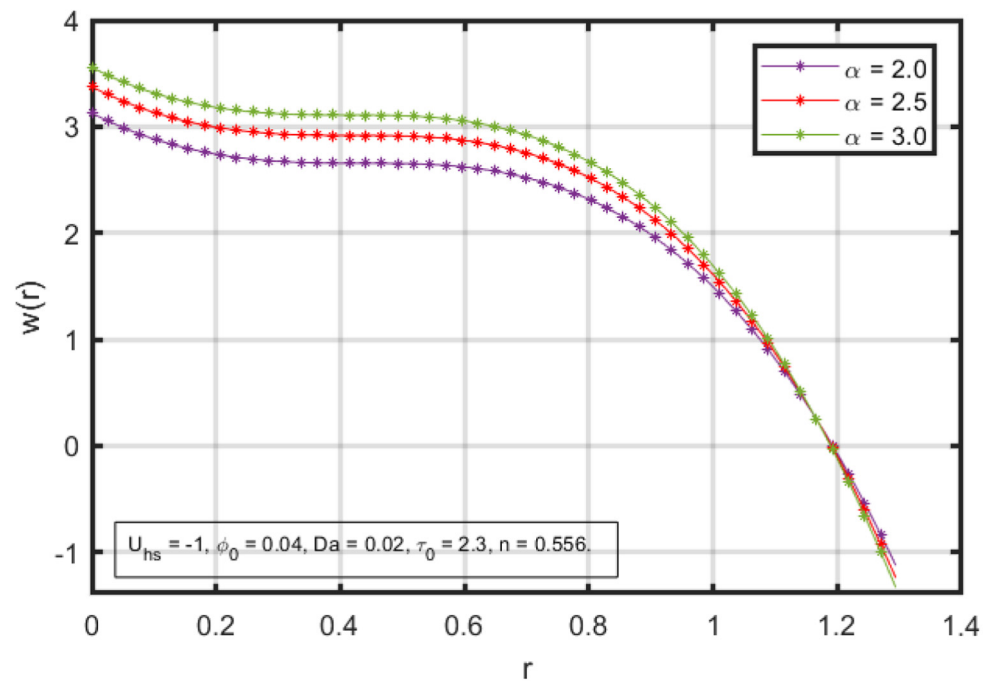
3. RESULTS AND DISCUSSION

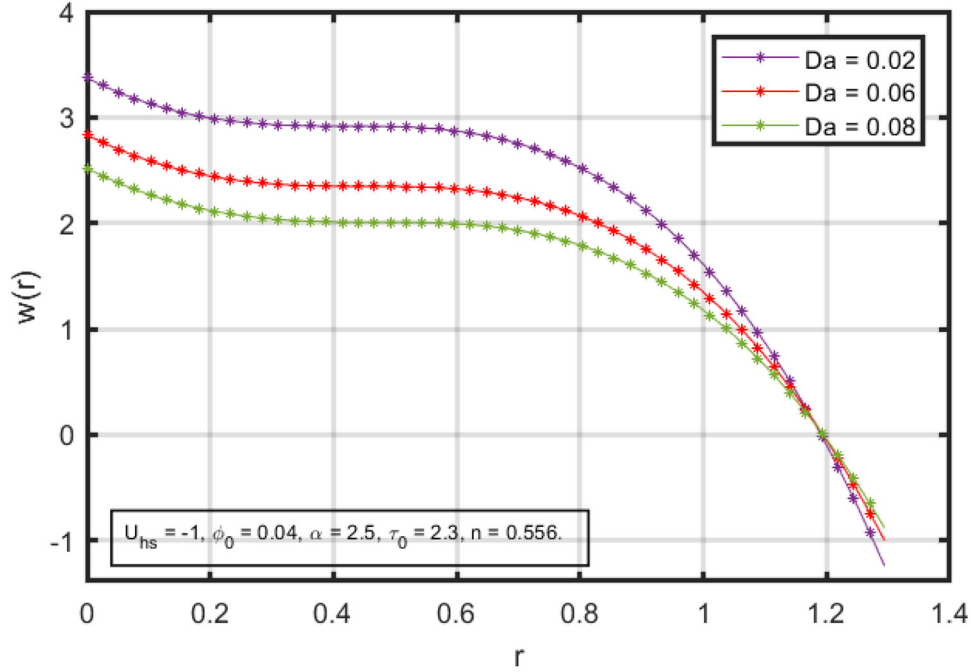
In this portion, the behavior of various physical properties of the fluid flow such as velocity field, pressure gradient, frictional force, Nusselt number, streamline pattern, and temperature is examined subject to multiple values of involved parameters. The thermophysical characteristics of both nanoparticles and drilling fluid calculated at the temperature of 322.13K are given in Table 1. Here a temperature difference of about 20 °C is assumed within the fluid flow and ZnO nanoparticles with an average diameter of 30 nm are scattered in the drilling fluid. The outer diameter of the drill pipe is chosen to be 0.05 m. Based on the properties described in Table 1 and for values of Herschel-Bulkley fluid parameters given in [12] for drilling fluids, the Prandtl number of Aphron based drilling fluid at 49 °C is 5.89, the dimensionless thermophoretic parameter is $N_t = 1.0981 \times 10^{-5}$, and the Brownian diffusion parameter is $N_b = 8.8052 \times 10^{-7}$ which may vary by the variations in any of the involved parameters such as nanoparticle volume fraction and consistency index.

3.1. Velocity profiles

The alterations in the velocity profile along the radial axis for different values of embedded parameters are displayed in Figures 2–6. Figure 2 describes the impression of the power-law index and yield stress on the velocity profile. It is evident that the fluid flow is accelerated for larger n and τ_0 . As for increment in n , the yield stress parameter increases, and the consistency index for aphron drilling fluid decreases which results in greater fluid velocity. The impact of the electroosmotic velocity parameter on the axial velocity distribution is depicted in Figure 3. Here the negative values of the electroosmotic parameter signify positively oriented electric forces whereas, the positive values imply the reversed direction of the applied electric field. It can be seen from the


 FIGURE 2. Velocity plot *versus* radial distance for power-law index.

 FIGURE 3. Velocity plot *versus* radial distance for electroosmotic velocity parameter.

FIGURE 4. Velocity plot *versus* radial distance for nanoparticle volume fraction.FIGURE 5. Velocity plot *versus* radial distance for slip velocity parameter.


 FIGURE 6. Velocity plot *versus* radial distance for permeability number.

resulting figure that the velocity field is maximum for negative U_{hs} as in this case the flow due to electroosmosis contributes to the flow by peristalsis. In the absence of an electric field *i.e.*, for $U_{hs} = 0$, flow occurs only due to peristaltic propulsion. However, for positive values of U_{hs} , a backflow occurs due to reversal of electric field, and overall axial velocity declines. In Figure 4, the velocity field is plotted for varying values of nanoparticle volume fraction. The resulting graph infers that the velocity of the fluid tends to enhance for a larger fraction of nanoparticles. Although the viscosity of the fluid grows by increasing the concentration of nanoparticles, there is an increment in the pressure gradient force which eventually raises the velocity. The trends in velocity distribution for varying choices of velocity slip parameter are illustrated in Figure 5. An increasing trend in axial velocity is witnessed for growing values of slip parameter. Physically presence of stronger slip at tube walls results in less adherence of fluid with pipe walls which tends to expedite the flow phenomenon. Figure 6 is prepared to investigate the effect of the porosity parameter on the velocity curves. It can be clearly observed that a larger porosity parameter produces a diminution in the velocity profile. A similar impact of permeability number on the velocity profile can be seen through [7].

3.2. Trapping phenomenon

One of the most important properties associated with the peristaltic flow is trapping in which the splitting of streamlines happens to encase some fluid which results in the circulatory flow pattern within the flow domain. The circulatory flow patterns are also termed as fluid boluses which travel along with the peristaltic wave. The outcomes of electroosmotic velocity parameter on trapping phenomenon are investigated through Figure 7a–c. The resulting figures indicate that for an electric field in the positive axial direction, the volume of trapping bolus is considerably larger than the trapped volume in case of no electric field *i.e.*, $U_{hs} = 0$, and in case of reversal electric field *i.e.*, $U_{hs} > 0$. As the opposite axial electric field slows down the fluid flow process and streamlines cannot properly circulate, therefore, the circulatory volume shrinks in this case. The impact of the power-law index on the trapping phenomenon is visualized through Figure 8a–c. As larger values of n correspond to a decline in the consistency index of the drilling fluid, therefore, the volume of the circulation within the

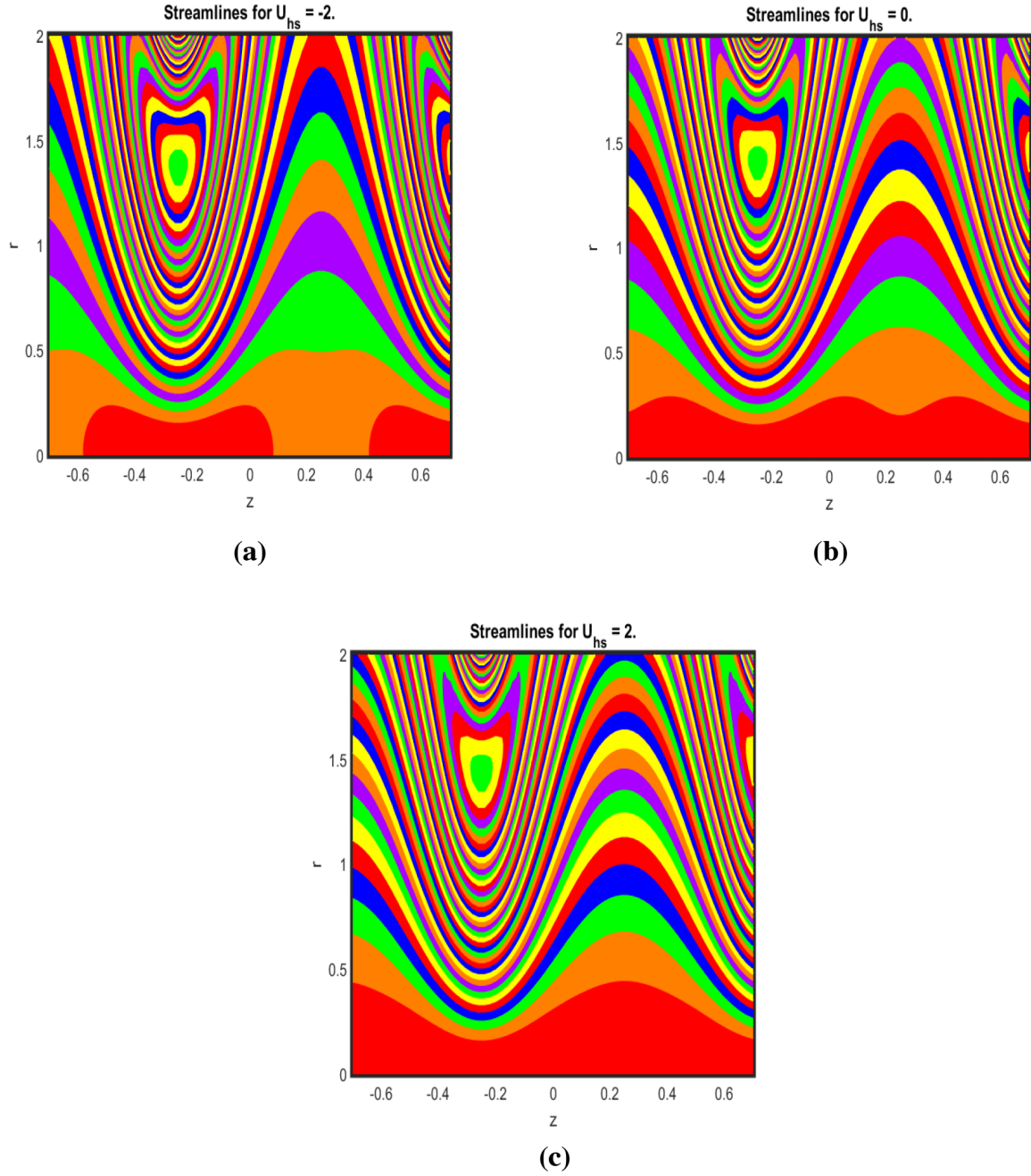


FIGURE 7. Circulatory flow pattern for electroosmotic velocity parameter.

flow field rises. Figure 9a–c portray the streamline pattern for multiple choices of velocity slip parameter. For larger slip velocities, the fluid in the vicinity of pipe walls is no more at rest relative to peristaltic waves which result in the circulation of streamlines to a larger extent and hence volume of the trapped bolus extends significantly.

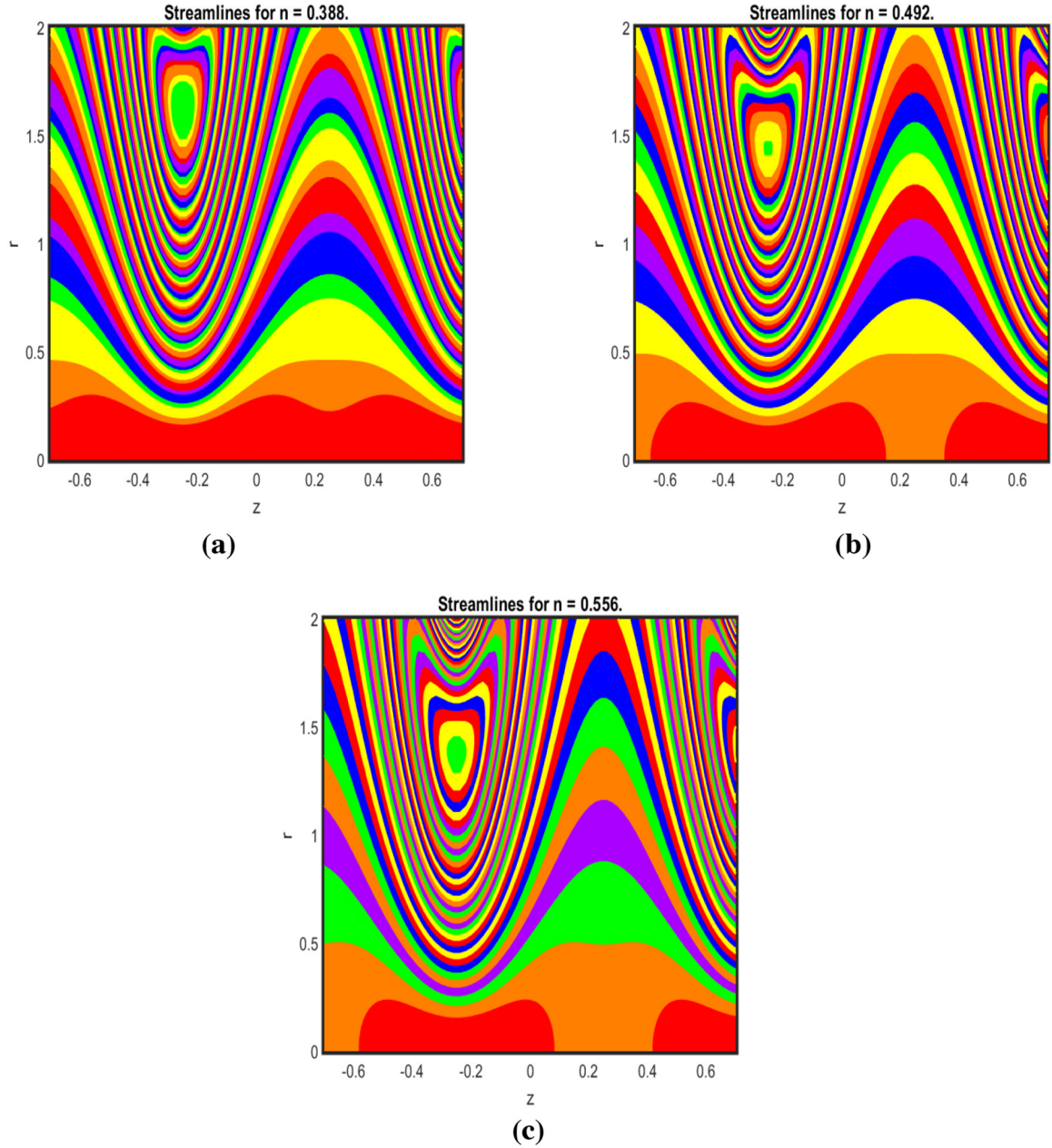


FIGURE 8. Circulatory flow pattern for power-law index.

3.3. Pressure gradient profile

In order to forecast the pumping characteristics of a peristaltic pump, it is essential to investigate the amount of pressure force induced per wavelength of the sinusoidal wave as it is the most important driving force in such kinds of flow. Figures 10–14 are presented to demonstrate the role of various controlling parameters on the pumping performance of the model. The pressure rise per wavelength is drawn against the flow rate for diversified values of parameters for both assisting and retarding electroosmotic forces. It can be concluded

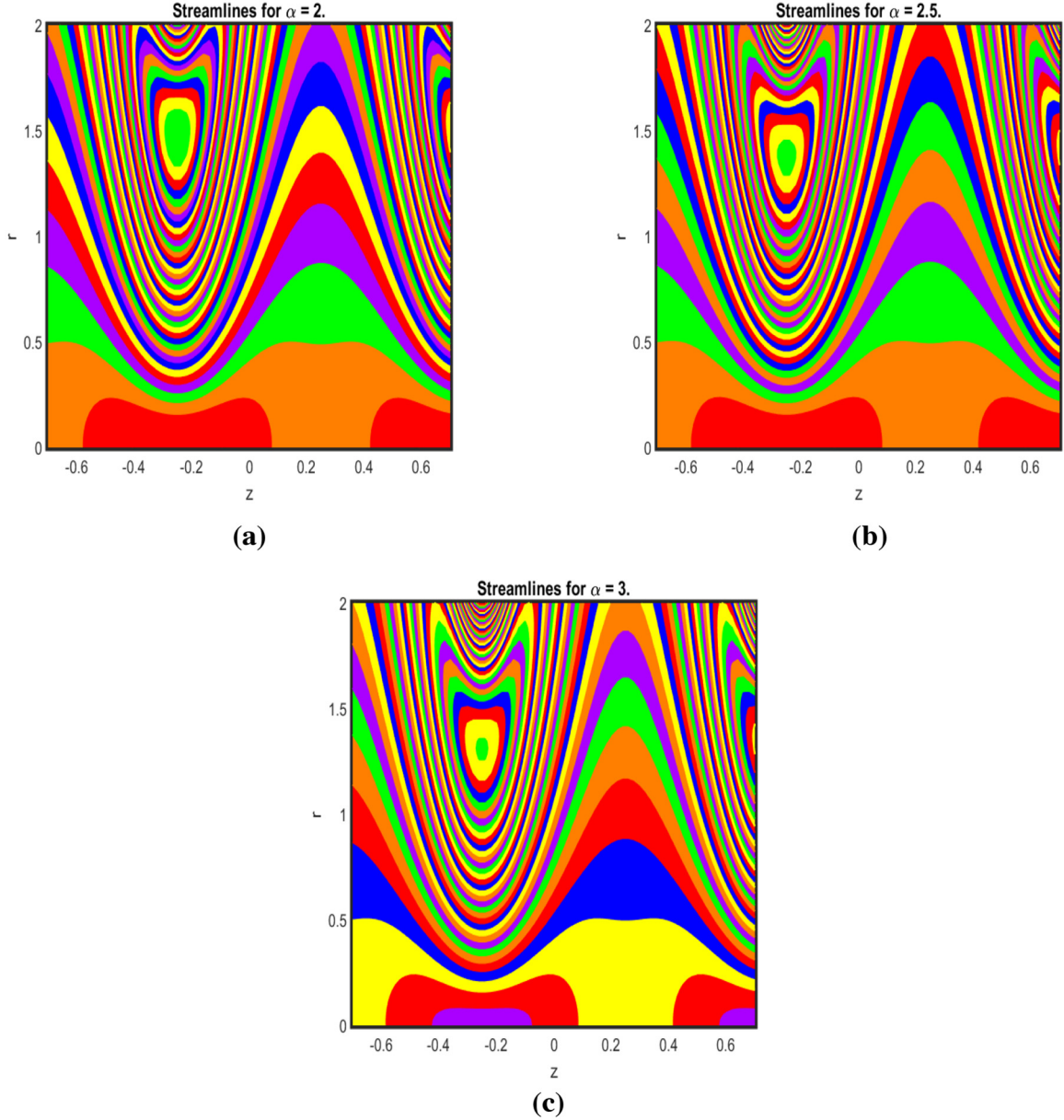
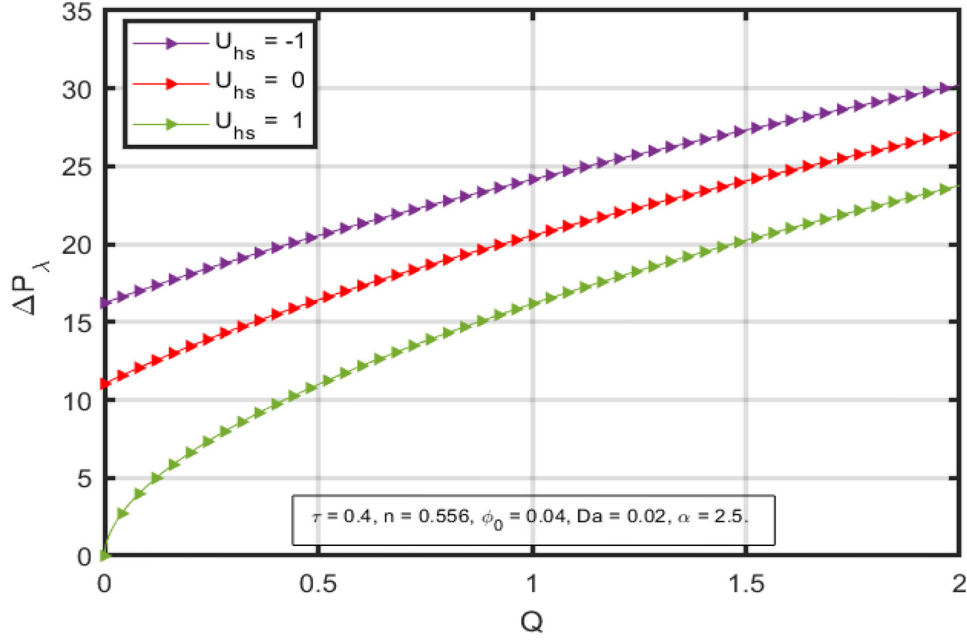


FIGURE 9. Circulatory flow pattern for the slip parameter.

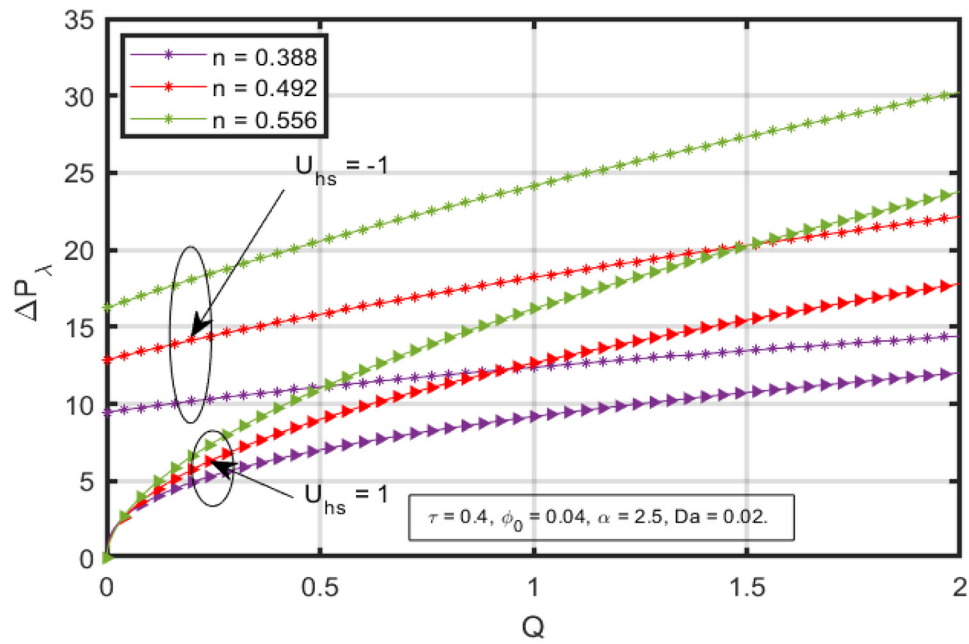
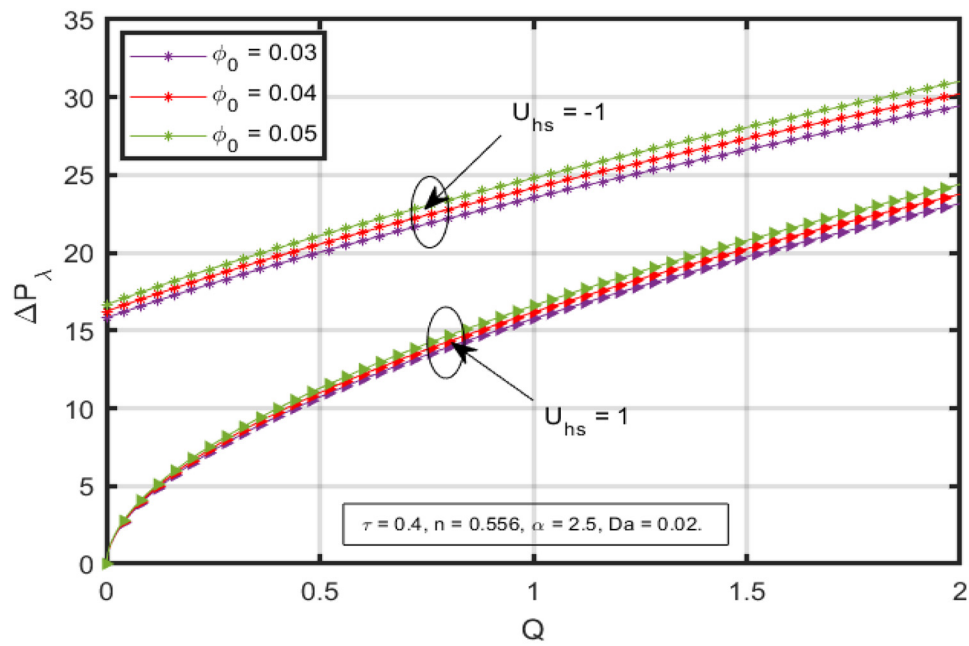
that pressure rise is directly associated with the flow rate. Figure 10 depicts the impact of the electroosmotic velocity parameter on ΔP_λ . It is found that pressure rise per wavelength is maximum for positively directed electroosmosis and is minimum in the case of opposite electroosmosis. The case of pure peristaltic pumping lies in between the above discussed cases. Clearly, the electric field oriented along a positive z -direction favors the flow due to the propagation of waves and consequently, a stronger pressure force is generated. For the reversed electric field, the assisting pressure gradient by the peristaltic wave is diminished by the opposite electroosmotic velocity. Therefore, pressure rise, in this case, is lower than the case of pure peristaltic pumping *i.e.*, $U_{hs} = 0$. The alterations in the pressure rise for alternating values of the power-law index are computed in Figure 11 for

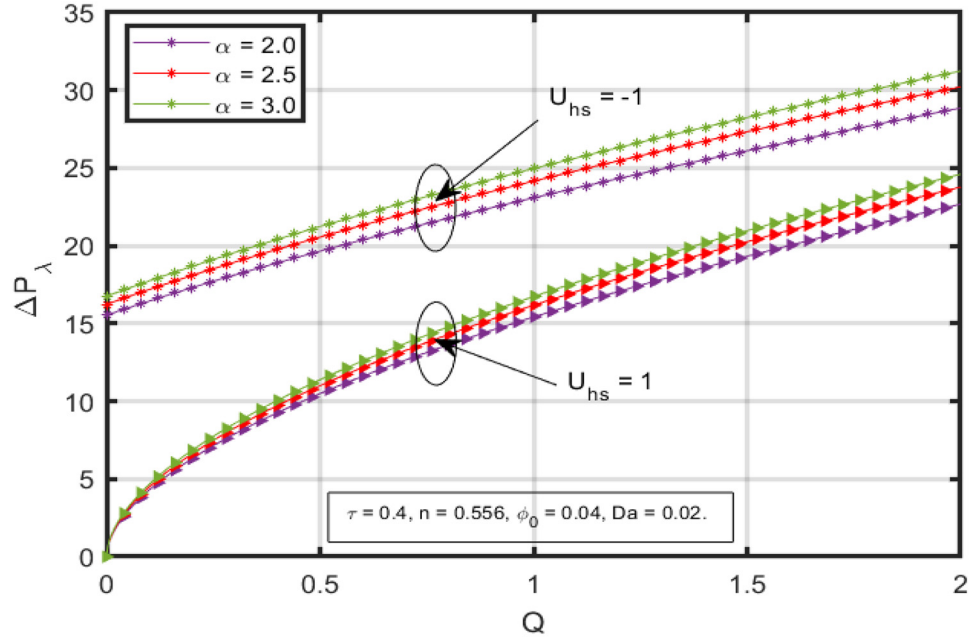
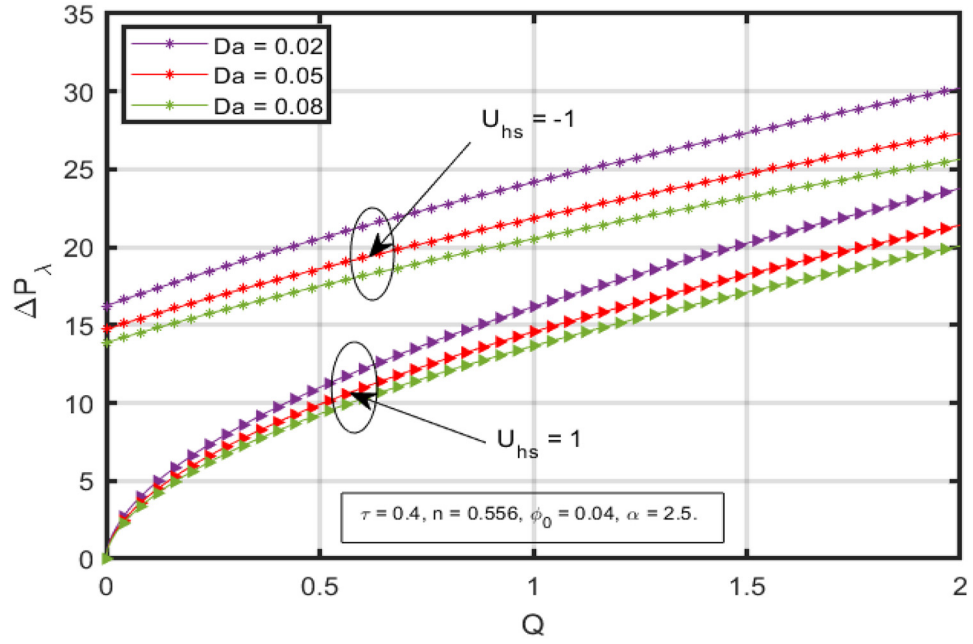

 FIGURE 10. ΔP_λ versus Q for electroosmotic velocity parameter.

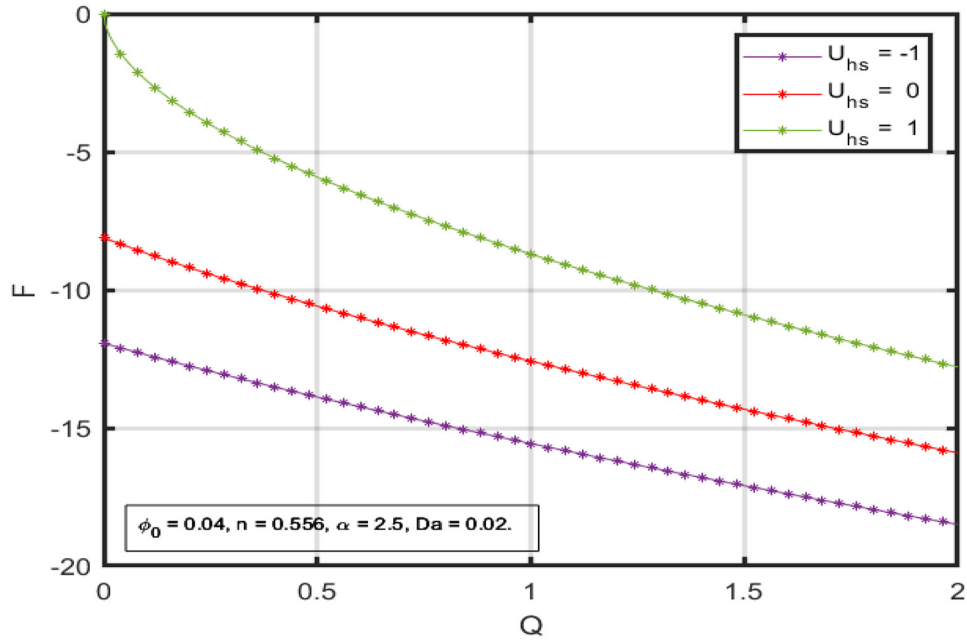
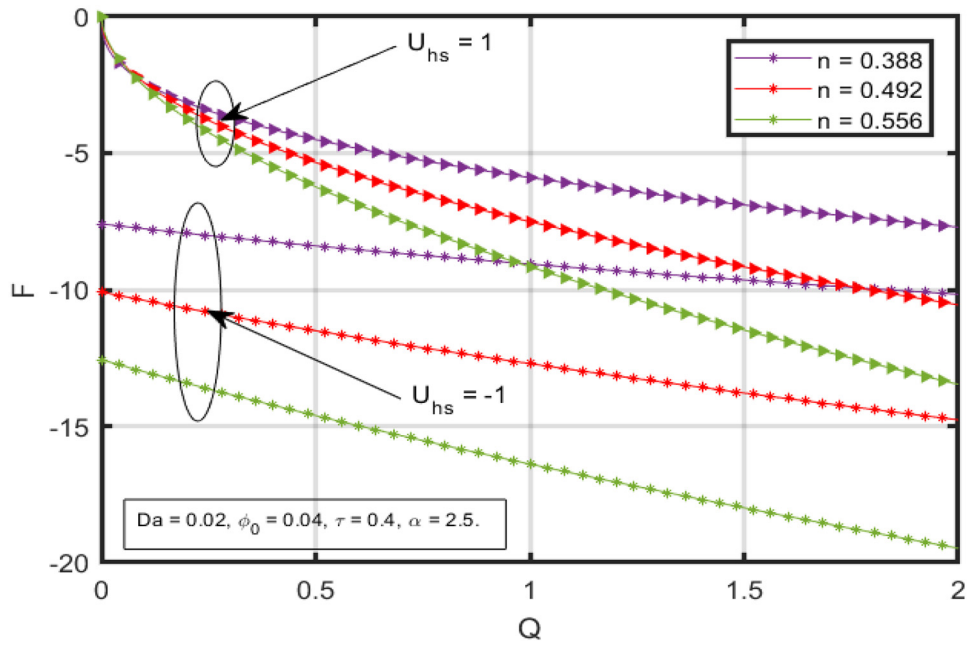
both assisting and retarding electric fields. The magnitude of pressure rise per wavelength tends to enhance for the rising values of n in both cases. Although there is a slight decrease in the shear-thinning properties of fluid for larger n , there is a considerable suppression in the viscosity of the drilling fluid which aids the generation of stronger pressure force across the pipe. The effect of nanoparticle volume fraction on pumping performance is examined in Figure 12. An elevation in pressure rise is observed for a larger volume fraction of nanoparticles. Figure 13 contains the pressure rise curves for varying values of velocity slip parameter. The presence of larger slip velocities in the vicinity of pipe walls physically means that fluid in the surrounding of walls is no more at rest relative to the boundaries. These slip velocities of fluid particles cause an uplift in the pressure rise over one wavelength. The outcomes of the permeability parameter on pressure rise per wavelength are revealed in Figure 14. It is found that pressure rise is inversely proportional to the permeability parameter.

3.4. Frictional force

The dimensionless frictional force at a pipe wall across one wavelength is calculated by numerical integration of equation (2.41) and the impacts of various physical parameters of interest on the frictional force are displayed in Figures 15–19. In all these figures, it is observed that the frictional force shows the opposite trend *versus* the volume rate. Figure 15 portrays the influence of the electroosmotic velocity parameter on F . It is found that more resistance is offered to the fluid flow for altering the direction of the electric field from negative to positive direction. It is a quite justifiable result as more collision of fluid particles occurs in case of assisting electric field which increases the magnitude of frictional forces. To figure out the influence of the power-law index on the frictional force, Figure 16 is plotted. The curves are drawn for both assisting and resisting electric fields. It is revealed that the magnitude of the frictional force is significantly larger in the case of the assisting electric fields as compared to the case of opposing electric fields. Moreover, the magnitude of frictional force is directly associated with the power-law index. As viscous resistive forces become stronger for a larger power-law index due to a decrease in the shear-thinning aspects of the considered fluid, therefore, the magnitude of frictional force increases. The outcomes of nanoparticle volume fraction on the frictional force distribution are depicted in Figure 17. The intensity of frictional forces increases with the insertion of more nanoparticles in the base fluid.

FIGURE 11. ΔP_λ versus Q for power-law index.FIGURE 12. ΔP_λ versus Q for nanoparticle volume fraction.


 FIGURE 13. ΔP_λ versus Q for slip velocity parameter.

 FIGURE 14. ΔP_λ versus Q for permeability number.

FIGURE 15. Frictional force *versus* flow rate for electroosmotic velocity parameter.FIGURE 16. Frictional force *versus* flow rate for power-law index.

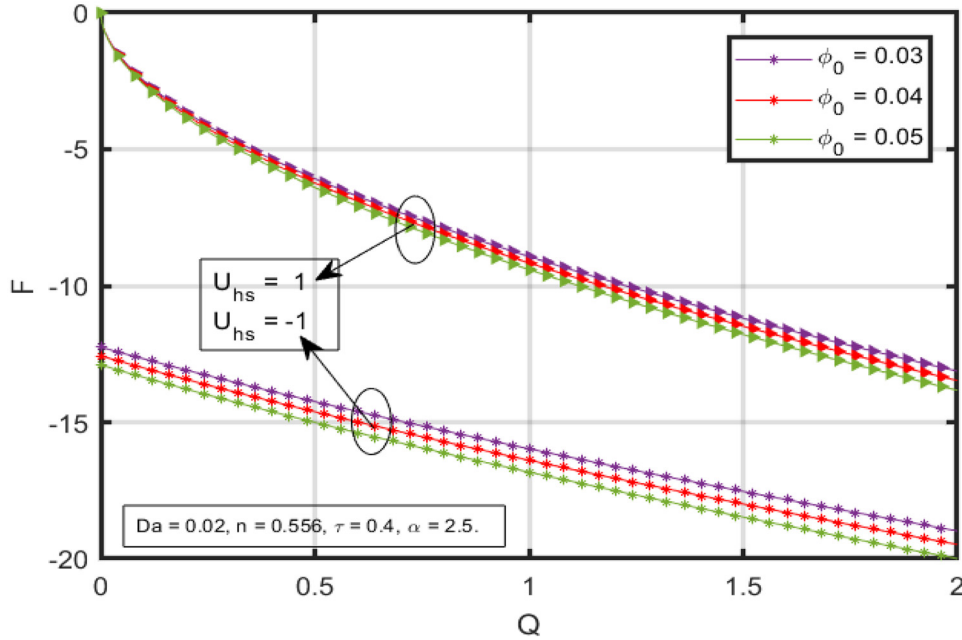
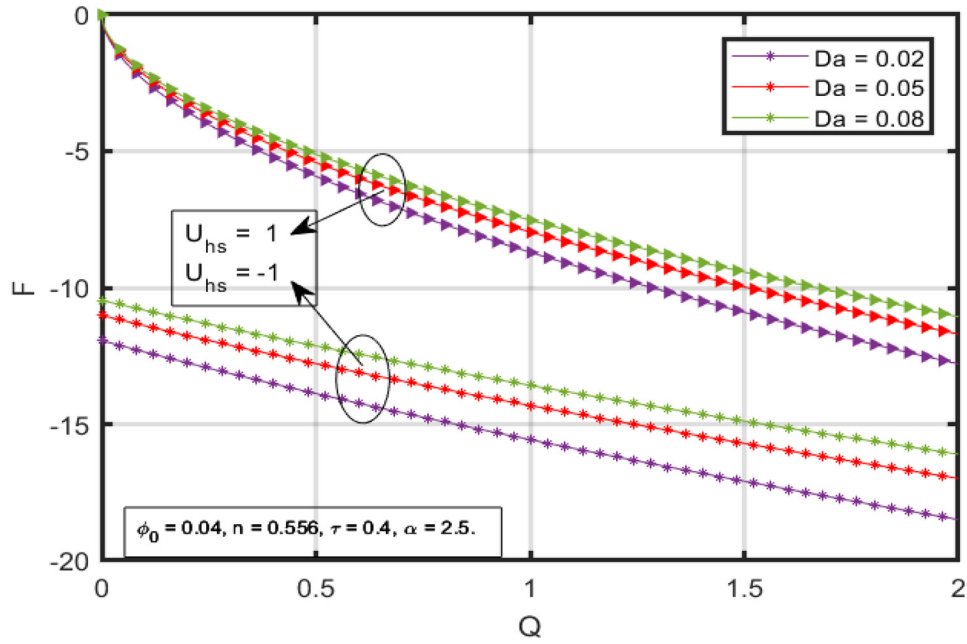
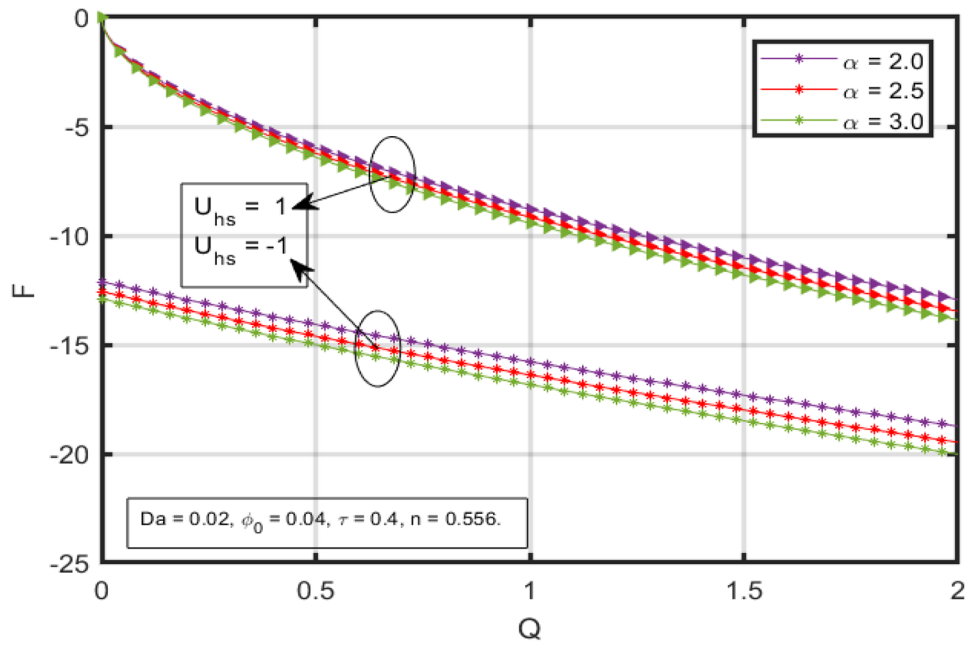
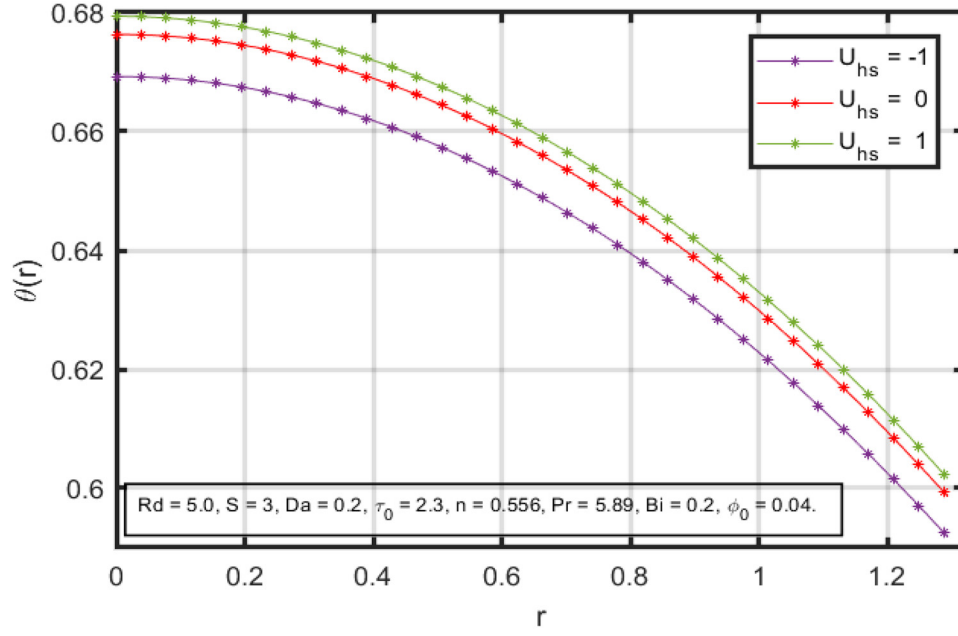
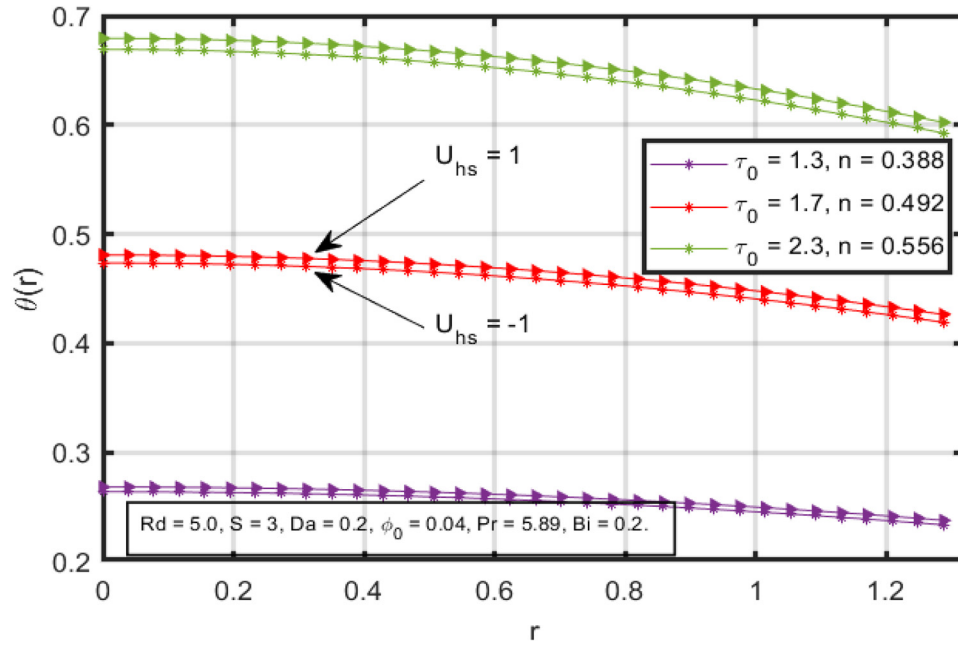

 FIGURE 17. Frictional force *versus* flow rate for power-law index.

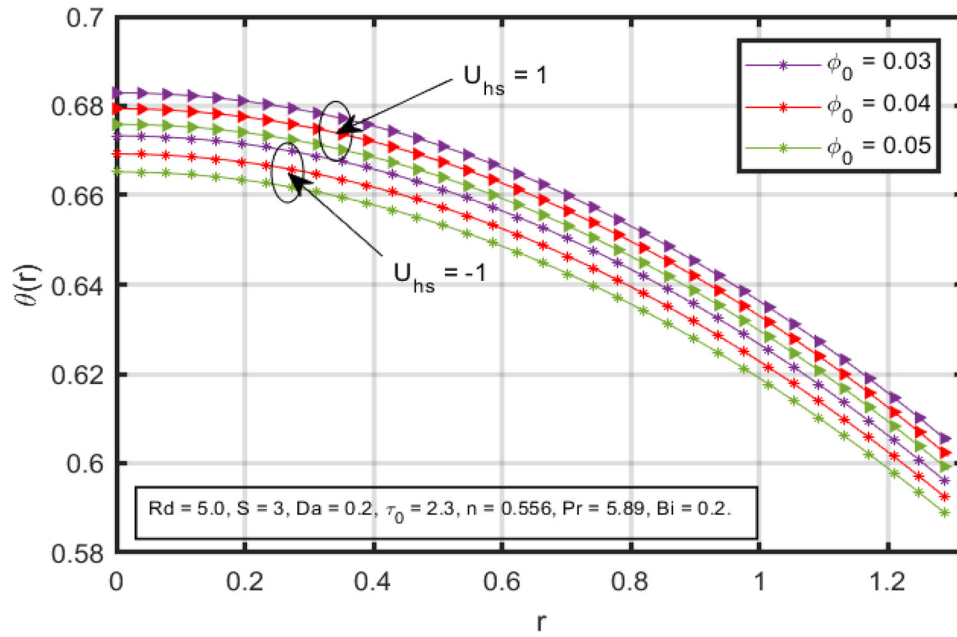
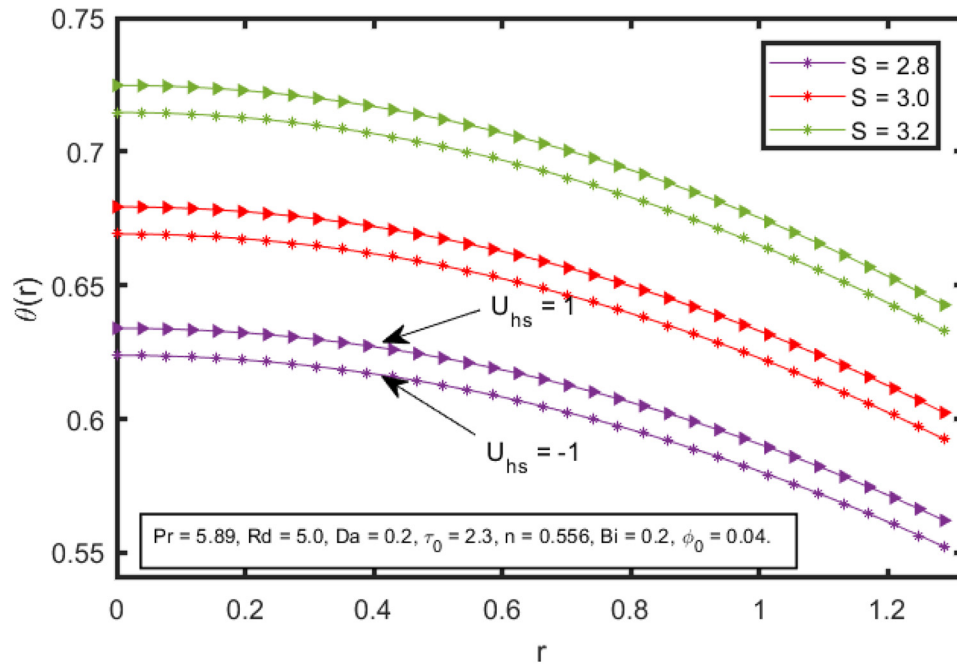
Figure 18 portrays the friction force curves for various values of permeability number. There is a reduction in the strength of frictional forces for growth in permeability number. To predict the effectiveness of the velocity slip parameter of the frictional force, Figure 19 is sketched. There is an increment in the magnitude of frictional forces for larger slip parameters.

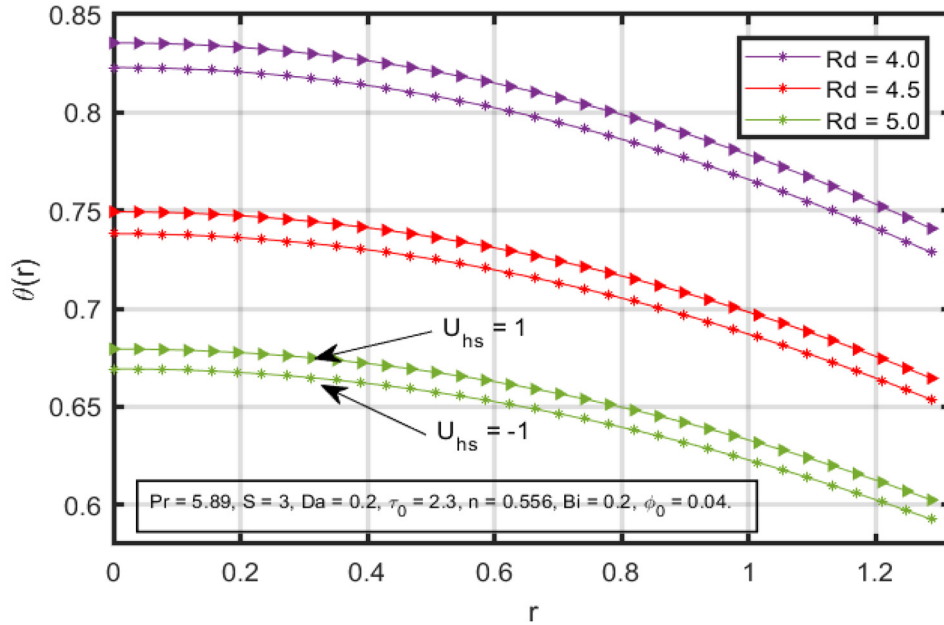
3.5. Temperature profiles

The consequences of various controlling parameters on the thermal behavior of drilling nanofluid are addressed in Figures 20–25. The temperature profile of the nanofluid helps us to judge the thermal efficiency of the working fluid under different circumstances. The impression of electroosmotic velocity parameters on the temperature is noticed in Figure 20. It is apparent that when the electric field is applied in such a way that it assists the fluid flow, the temperature of the fluid is minimum. However, for an oppositely directed electric force, the temperature of the nanofluid rises. This result is quite justifiable as for the backward electroosmotic velocity, fluid particles moving in a forward direction experience a drag due to back electroosmosis which raises the temperature. Figure 21 demonstrates the influence of the power-law index and yield stress parameter on temperature profiles. There is an intensification in the temperature of nanofluid for larger n . Here the temperature curves are plotted for both directions of electroosmotic velocity. It is found that curves for the negative values of the electroosmotic velocity parameter lie slightly lower than the curves for positive U_{hs} . The alterations in the thermal field *via* growing ϕ_0 are captured in Figure 22. As increasing the concentration of nanoparticles in the nanofluid raises the cooling tendency of the fluid, therefore, the temperature of the fluid drops down. Figure 23 portends the evolution in the temperature profile subject to the multiple values of S . As the Joule heating parameter is the measure of the rate at which heat energy is generated due to the passage of electric current through a fluid medium, therefore, a significant enhancement in temperature occurs for the larger Joule heating parameter. In Figure 24, temperature curves are plotted for the enhancing values of the radiation parameter. As R_d is inversely associated with the mean absorption capability of the fluid, the advancement in the radiation parameter indicates that less heat is absorbed by the nanofluid which causes a decline in the temperature.

FIGURE 18. Frictional force *versus* flow rate for permeability number.FIGURE 19. Frictional force *versus* flow rate for slip velocity parameter.


 FIGURE 20. Thermal distribution against r for the electroosmotic velocity parameter.

 FIGURE 21. Thermal distribution against r for the power-law index.

FIGURE 22. Thermal distribution against r for the nanoparticle volume fraction.FIGURE 23. Thermal distribution against r for the Joule heating parameter.


 FIGURE 24. Thermal distribution against r for the radiation parameter.

3.6. Nusselt number

The impact of numerous intricate parameters on Nusselt number distribution can be visualized through Figures 25–29. Figure 25 reveals that the Nusselt number significantly rises for augmentation in the power-law index and yield stress parameter. An increment in n corresponds to the weaker viscous forces which accelerates the convective heat transfer of the fluid. As the Nusselt number is in direct relationship with the convective heat transfer, therefore, there is a significant increase in the magnitude of the Nusselt number. The amendments in the Nusselt number for diversified values of the Joule heating parameter are depicted in Figure 26. It is inferred that there is remarkable growth in the Nusselt number *via* rising S . An uplift in S physically means that a stronger electric field is applied across the pipe ends which stimulates the process of electroosmosis. Therefore, convective heat transfer becomes more effective compared to conductive heat transfer, and hence, the Nusselt number increases. The magnitude of the Nusselt number declines *via* an increase in the fraction of nanoparticles in the base fluid as seen in Figure 27. The modifications in the Nusselt number subject to modification in radiation parameter are manifested in Figure 28. It is anticipated that the Nusselt number drops upon increasing the radiation parameter. In Figure 29, the Nusselt number is plotted for increasing the temperature difference within the fluid. It is revealed that the Nusselt number escalates for higher temperature difference and this increment is more significant at the wave trough when compared with the behavior of the Nusselt number at the wave crest. The reason behind this impression is that the convective flow of heat energy is boosted when there is a larger temperature difference within the moving fluid.

4. SUMMARY AND CONCLUSIONS

In this framework, a mathematical model is worked out to investigate the flow of aphron-based drilling nanofluid through a pipe. The viscoplastic characteristics of the fluid are predicted by the Herschel-Bulkley fluid model and the values of involved parameters are used from [12] where these parameters are calculated for aphron drilling fluid experimentally. The fluid flow is driven by peristalsis and electroosmotic forces. The existence of viscous dissipation, porous medium, wall slip velocity, and Joule heating is also considered in the

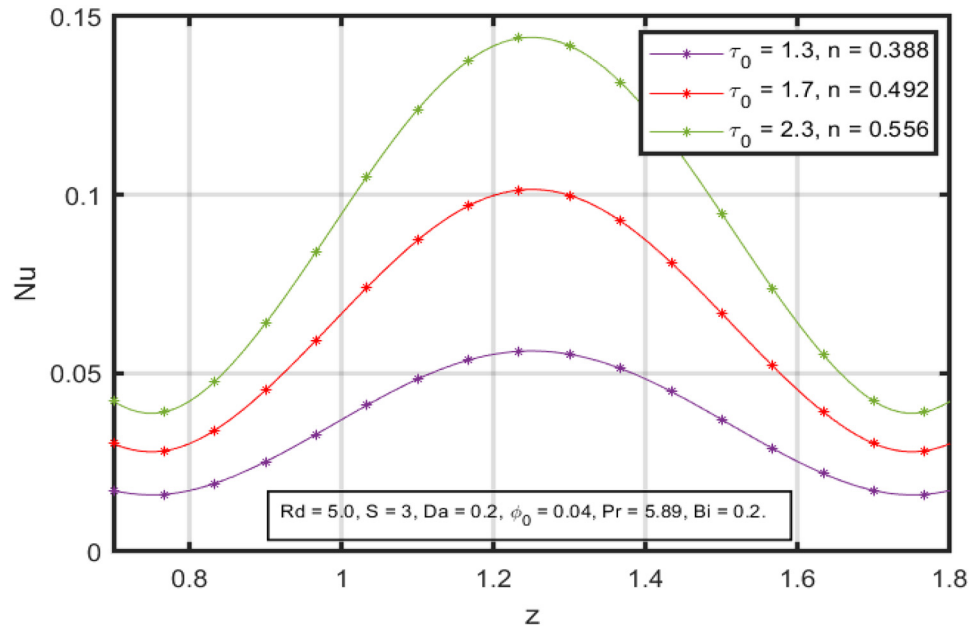


FIGURE 25. Nusselt number for the power-law index.

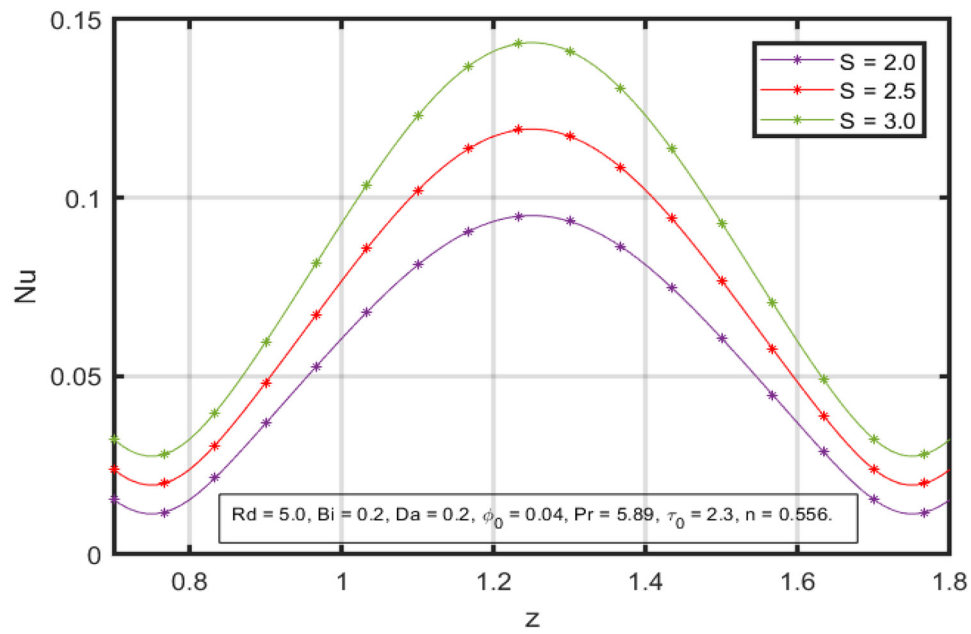


FIGURE 26. Nusselt number for the Joule heating parameter.

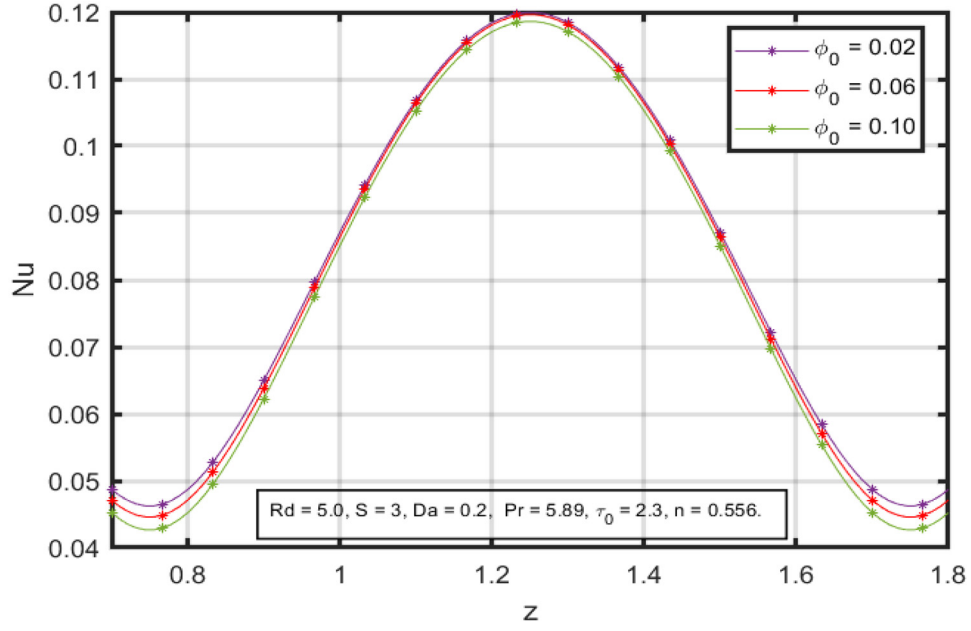


FIGURE 27. Nusselt number for the nanoparticle volume fraction.

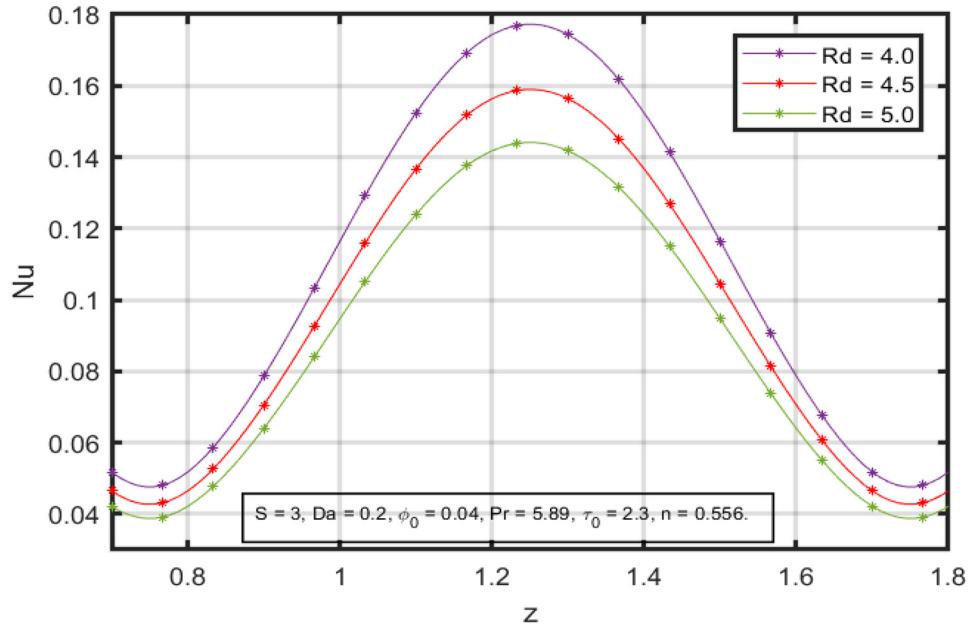


FIGURE 28. Nusselt number for the radiation parameter.

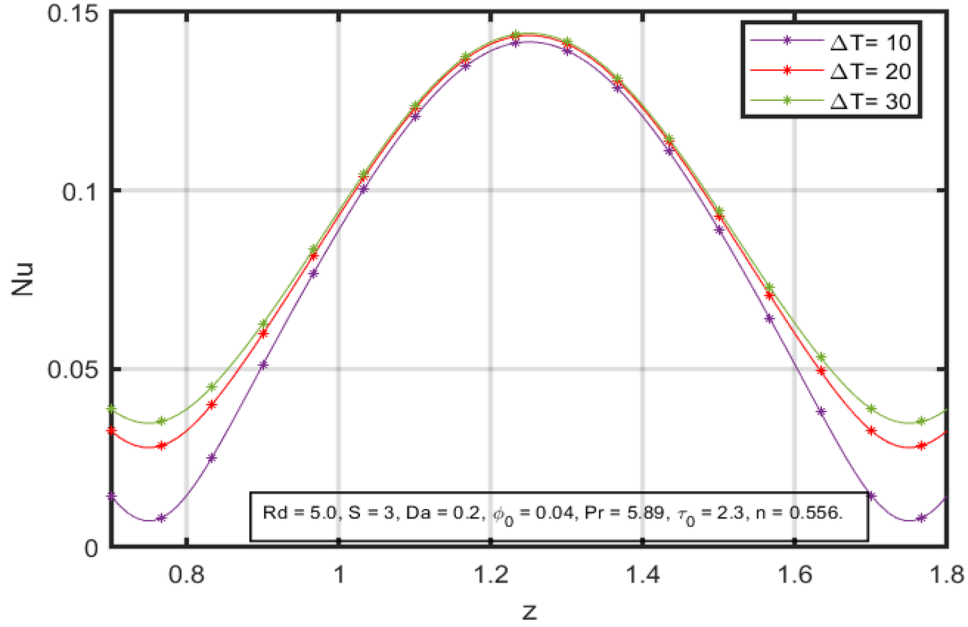


FIGURE 29. Nusselt number for the temperature difference.

analysis of the heat transfer phenomenon. The numerical solution of nonlinear coupled equations is computed through the bvp4c package of mathematical software MATLAB. The key findings of the analysis are listed as:

- The electroosmotic velocity parameter remarkably accelerates the fluid flow and produces a decay in the temperature of the fluid when the electric field is applied in a forwarding direction. So, it can be concluded that the process of electroosmosis contributes a lot to the efficiency of the pump and hence, helps in controlling the fluid flow.
- It is found that increasing the fraction of nanoparticles tends to raise the fluid velocity and suppresses the temperature profile of the nanofluid.
- This investigation reveals that an increment in the power-law index reduces the effectiveness of frictional forces and substantially improves the convective heat transfer tendency of the drilling fluid.
- The magnitude of the Nusselt number significantly rises for an increase in the power-law index and yield stress of the fluid.
- The presence of larger slip velocities at the wall boosts the fluid acceleration and the circulatory flow pattern within the flow field.
- The pressure generated across one wavelength can be raised and frictional force can be reduced by raising the intensity of the applied electric field in the positive axial direction.

NOMENCLATURE

Symbols	Description	Unit
b_0	The radius of the pipe	m
λ	Wavelength parameter	m
a	Wave amplitude	m
$\tilde{U}, \tilde{W}$	Radial and axial velocity component	ms^{-2}

$\rho_{nf}, \rho_f, \rho_s$	The density of the nanofluid, base fluid, and nanoparticles.	Kg m^{-3}
$\check{t}$	Time parameter	S
$\check{P}$	Pressure field	Nm^{-2}
$\check{\tau}_{\check{R}\check{R}}, \check{\tau}_{\check{R}\check{Z}}, \check{\tau}_{\check{Z}\check{Z}}$	Shear stress components	Nm^{-2}
ρ_e	Local electric charge density	C m^{-3}
$E_{\check{Z}}$	The axial electric field	$\text{Kg m s}^{-3} \text{A}^{-1}$
C_{nf}, C_s	Specific heat of nanofluid and nanoparticles	J/Kg K
$\check{T}$	Temperature field	K
D_B	Brownian diffusion parameter	$\text{m}^2 \text{s}^{-1}$
$\check{\Phi}$	Concentration field	Kg m^{-3}
σ_{nf}	Electrical conductivity of nanofluid	S/m
σ^*	Stefan Boltzmann constant	$\text{W m}^{-2} \text{K}^{-4}$
$D_{\check{T}}$	Thermophoretic parameter	$\text{m}^2 \text{s}^{-1}$
κ^*	Mean absorption coefficient	m^{-1}
A_1	Rivlin Erickson tensor	s^{-1}
τ^*	Yield stress	Nm^{-2}
K	Consistency index	Pa.s^n
ϕ_0	Nanoparticle volume fraction	$[-]$
k_{nf}, k_f, k_s	Thermal conductivity of the nanofluid, base fluid, and nanoparticles.	$\text{W m}^{-1} \text{K}^{-1}$
d_p	The diameter of nanoparticles	M
T_{fr}	The freezing temperature of the base fluid	K
$\check{E}$	Electric potential	V
ε_0	The dielectric constant for vacuum.	F m^{-1}
ε_m	The relative permittivity of the fluid medium	$[-]$
n_0	The bulk concentration of ionic species	kg m^{-3}
e	Charge of electron	C
z	Valence	$[-]$
p	Dimensionless pressure parameter	$[-]$
Da	Darcy parameter	$[-]$
U_{hs}	Electroosmotic velocity parameter	$[-]$
Rd	The dimensionless radiation parameter	$[-]$
Ec	The Eckert number	$[-]$
n	Power-Law index	$[-]$
ϵ	Amplitude ratio	$[-]$
ψ	Dimensionless stream function	$[-]$
Br	Brinkmann number	$[-]$
ΔT	Temperature difference parameter	$[-]$
Pr	Prandtl number	$[-]$
N_b	Dimensionless Brownian diffusion parameter	$[-]$
α	Non-dimensional slip parameter	$[-]$
N_t	Dimensionless thermophoretic parameter	$[-]$
Bi	Thermal Biot number	$[-]$
S	The Joule heating parameter	$[-]$
ξ	Dimensionless zeta potential	$[-]$
F	Dimensionless frictional force	$[-]$
Nu	Nusselt number	$[-]$
Q	The dimensionless averaged flux	$[-]$
θ	Dimensionless temperature	$[-]$
Φ	The dimensionless nanoparticle volume fraction	$[-]$

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